

Basic Humidity Definitions

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Abstract

This Workshop presents a tutorial on the different measures of humidity and their relationships to each other. Air, or moist air as it is called, consists of water vapor and the remaining dry air. The tutorial starts with the conceptually simplest measures of humidity involving masses and volumes of water vapor, dry air, and moist air. Although simple conceptually, these measures are impractical to implement. After introducing the Ideal Gas Law, saturation vapor pressure and enhancement factors are presented. Then other measures of humidity are introduced involving the moles of the components of moist air. Finally, the NIST two-temperature two-pressure humidity generator is discussed.

Mixing Ratio by Weight

From Wexler⁽¹⁾, in a given sample of moist air, the Mixing Ratio by Weight is

$$\frac{\text{mass of water vapor}}{\text{mass of dry air}}$$

The Mixing Ratio by Weight of a moist air sample does not change when either temperature or pressure changes. The Mixing Ratio by Weight is related to the Mixing Ratio by Volume through the Molecular Weights of water vapor and dry air.

$$\text{mixing ratio by weight} = \frac{M_v}{M_a} \times \text{mixing ratio by volume}$$

Specific Humidity

From Wexler⁽¹⁾, in a given sample of moist air, the Specific Humidity is

$$\frac{\text{mass of water vapor}}{\text{total mass of moist air}}$$

The Specific Humidity of a moist air sample does not change when either temperature or pressure changes. In terms of Partial Pressures and Molecular Weights,

specific humidity

$$\begin{aligned} &= \frac{\text{mass of water vapor}}{\text{mass of water vapor} + \text{mass of dry air}} \\ &= \frac{M_v \times \text{moles of water vapor}}{M_v \times \text{moles of water vapor} + M_a \times \text{moles of dry air}} \\ &= \frac{M_v \times \text{partial pressure of water vapor}}{M_v \times \text{partial pressure of water vapor} + M_a \times \text{partial pressure of dry air}} \\ &= \frac{M_v \times e}{M_v \times e + M_a \times (P - e)} \end{aligned}$$

Percent by Weight

From Wexler⁽¹⁾, the Percent by Weight of a given sample of moist air is the Specific Humidity expressed as a percent,

$$\frac{\text{mass of water vapor}}{\text{total mass of moist air}} \times 100$$

The Percent by Weight of a moist air sample does not change when either temperature or pressure changes. The Percent by Weight is related to the Percent by Volume through the Molecular Weights of water vapor and dry air.

$$\frac{1}{\text{percent by weight}} - 1 = \frac{M_a}{M_v} \left(\frac{1}{\text{percent by volume}} - 1 \right)$$

Parts per Million by Weight

The Parts per Million by Weight of a given sample of moist air is the Mixing Ratio by Weight expressed in parts per million,

$$\frac{\text{mass of water vapor}}{\text{mass of dry air}} \times 10^6$$

The Parts per Million by Weight of a moist air sample does not change when either temperature or pressure changes. The Parts per Million by Weight is related to the Parts per Million by Volume through the Molecular Weights of water vapor and dry air.

$$\text{parts per million by weight} = \frac{M_v}{M_a} \times \text{parts per million by volume}$$

Grains per Pound

In a given sample of moist air, the Grains per Pound is

$$\begin{aligned} \text{grains/lb} &= \frac{\text{grains of water vapor}}{\text{lb of moist air}} \\ &= \frac{\text{lb of water vapor}}{\text{lb of moist air}} \times 7000 \frac{\text{grains}}{\text{lb}} \end{aligned}$$

The Grains per Pound of a moist air sample does not change when either temperature or pressure changes. With the usual identification of weight and mass,

$$\text{grains/lb} = \text{specific humidity} \times 7000$$

Absolute Humidity, Vapor Concentration, Water Vapor Density

From Wexler⁽¹⁾, Absolute Humidity, Vapor Concentration, and Water Vapor Density are all the same. In a moist air sample, they all equal

$$\frac{\text{mass of water vapor}}{\text{volume of sample}}$$

Using the Ideal Gas Law, the Molecular Weight of water vapor, and Partial Pressures,

$$\text{absolute humidity} = \frac{M_v \times e}{R \times T}$$

where

M_v = molecular weight of water vapor

e = partial pressure of water vapor

R = Universal Gas Constant

T = Temperature in Kelvin

Dry Air Density

In a given sample of moist air, the Dry Air Density is

$$\frac{\text{mass of dry air}}{\text{volume of sample}}$$

Using the Ideal Gas Law, the Molecular Weight of dry air, and Partial Pressures,

$$\text{dry air density} = \frac{M_a (P - e)}{R \times T}$$

where

M_a = molecular weight of dry air

P = total pressure of sample

e = partial pressure of water vapor

R = Universal Gas Constant

T = Temperature in Kelvin

Moist Air Density

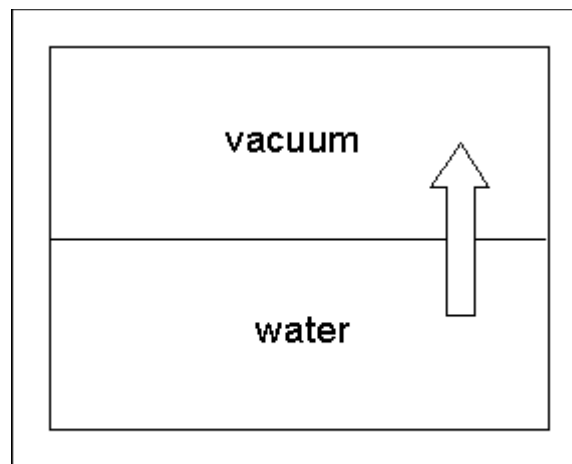
In a given sample of moist air, the Moist Air Density is

$$\frac{\text{mass of moist air}}{\text{volume of sample}}$$

Moist Air Density is the sum of Dry Air Density and Absolute Humidity.

$$\text{absolute humidity} + \text{dry air density} = \text{moist air density}$$

Saturation Vapor Pressure

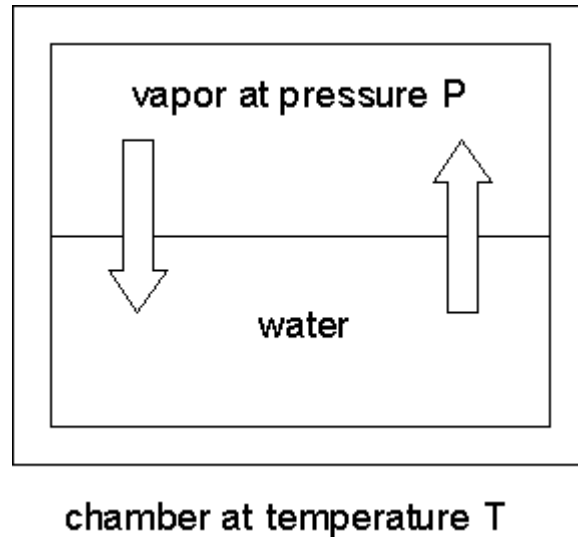


chamber at temperature T

Saturation Vapor Pressure is a function of temperature. Sonntag⁽²⁾ is one source of approximating formulas. The function can best be described by a lab setup. Imagine a chamber whose temperature T can be controlled. The chamber is partially filled with water. Initially, the

remaining space is a vacuum. The pressure P of the space over the water can be measured. At a fixed temperature, water molecules will leave the water and enter the space above at a fixed rate.

As water molecules accumulate over the liquid water, the pressure there will increase, and molecules will re-enter the liquid at an increasing rate. Finally, water molecules will be entering and leaving the liquid at the same rate, giving equilibrium and a constant pressure P over the water.



The equilibrium pressure P is the Saturation Vapor Pressure at temperature T .

$$e_s(T) = P$$

At temperatures above freezing, equilibrium is achieved over water. At temperatures below freezing, equilibrium can be achieved either over water or over ice. This gives two functions,

$$e_{ws}(T) \quad \text{and} \quad e_{is}(T)$$

The two functions agree for values of T above freezing. They differ for values of T below freezing.

Mole

A Mole is like a dozen or a gross, only much larger. From Wexler⁽¹⁾,

$$1 \text{ mole} = 6.023 \times 10^{26}$$

Molecular Weight

The Molecular Weight of a substance is the weight in grams of a mole of that substance. From Wexler⁽¹⁾,

$$\begin{aligned}\text{water vapor: } M_v &= 18.02 \text{ gm/mole} \\ \text{dry air: } M_a &= 28.9645 \text{ gm/mole}\end{aligned}$$

Mole Fraction

From Wexler⁽¹⁾, the Mole Fraction of a component gas present in a mixture of gases is

$$\frac{\text{moles of component gas}}{\text{total moles of mixture}}$$

From Dalton's Law,

$$\text{mole fraction} = \frac{\text{partial pressure of component gas}}{\text{total pressure of mixture}}$$

Universal Gas Constant

The Universal Gas Constant is denoted R. From Wexler⁽¹⁾,

$$R = 8.31432 \frac{\text{Joules}}{\text{mole} \times \text{Kelvin}}$$

Ideal Gas Law

The Ideal Gas Law relates the pressure, volume, moles, and temperature of a sample of ideal gas.

$$P \times V = n \times R \times T$$

where

P = pressure (Pascals)

V = volume (m^3)

n = moles

R = Universal Gas Constant $\left(\frac{\text{Joules}}{\text{mole} \times \text{Kelvin}} \right)$

T = temperature (Kelvin)

Dalton's Law

Dalton's Law states that the Ideal Gas Law applies to mixtures of ideal gases. Two ideal gases, n_1 moles of the first gas and n_2 moles of the second, both at the same temperature, must each individually satisfy the Ideal Gas Law.

$$P_1 \times V_1 = n_1 \times R \times T$$

$$P_2 \times V_2 = n_2 \times R \times T$$

If the two gases are combined to form a mixture of n moles, again at the same temperature, then

$$P \times V = n \times R \times T$$

$$n = n_1 + n_2$$

Since temperatures are all the same, the Mole Fraction of the first gas in the mixture must satisfy

$$\text{mole fraction} = \frac{n_1}{n} = \frac{P_1 \times V_1}{P \times V}$$

If, in addition to the temperatures, all three pressures are the same, then

$$\text{mole fraction} = \frac{n_1}{n} = \frac{V_1}{V}$$

If, in addition to the temperatures, all three volumes are the same, then

$$\text{mole fraction} = \frac{n_1}{n} = \frac{P_1}{P}$$

Similarly for the Mole Fraction of the second gas. Dalton's Law also applies to mixtures of more than two gases.

Partial Pressures

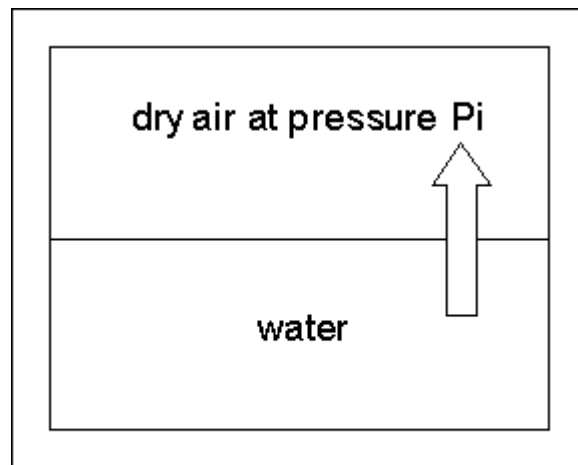
The pressure P of a mixture of gases is the sum of the Partial Pressures of the component gases.

$$P = P_1 + P_2 + P_3 + \dots$$

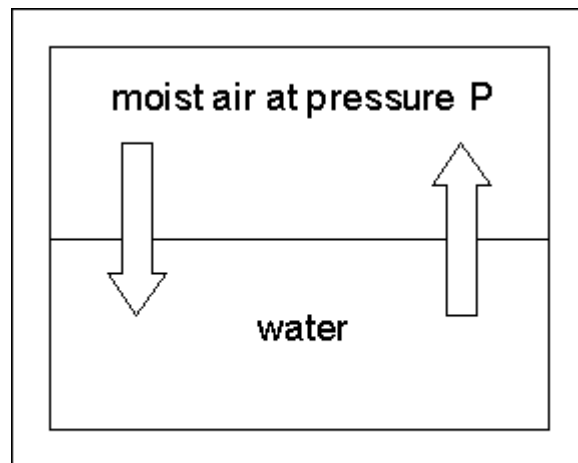
Where P_i , the Partial Pressure of the i th component gas, is the pressure that would be measured if the volume were occupied by only the i th component gas. This is a consequence Dalton's Law. In a sample of moist air, the total pressure is denoted P , and the Partial Pressure of the water vapor present is denoted e . Then the Partial Pressure of the dry air in the sample must be $(P-e)$.

Enhanced Saturation Vapor Pressure

Enhanced Saturation Vapor Pressure, or Saturation Vapor Pressure of Moist Air, is a function of both pressure and temperature. Like Saturation Vapor Pressure, it can best be described by a lab setup. Imagine a chamber whose temperature T and pressure can be controlled. The chamber is partially filled with water. The pressure of the space over the water can be measured. Initially, the remaining space is filled with dry air at initial pressure P_i . At a fixed temperature, water molecules will leave the water and enter the dry air above at a fixed rate.



chamber at temperature T



chamber at temperature T

As water molecules accumulate over the liquid water, the pressure there will increase, and molecules will re-enter the liquid at an increasing rate. Finally, water molecules will be entering and leaving the liquid at the same rate, giving equilibrium and a constant final pressure P over the water.

The Enhanced Saturation Vapor Pressure (Saturation Vapor Pressure of Moist Air) at temperature T and pressure P is the Partial Pressure due to the water vapor in the moist air.

$$e'_s(T,P) = P - P_i$$

At temperatures above freezing, equilibrium is achieved over water. At temperatures below freezing, equilibrium can be achieved either over water or over ice. This gives two functions, as with Saturation Vapor Pressure,

$$e'_{ws}(T,P) \quad \text{and} \quad e'_{is}(T,P)$$

The two functions agree for values of T above freezing. They differ for values of T below freezing.

Enhancement Factor

The Enhancement Factor at temperature T and pressure P is the ratio of the Enhanced Saturation Vapor Pressure to the Saturation Vapor Pressure. One source of approximating formulas is Greenspan⁽³⁾.

$$f(T,P) = \frac{e'_s(T,P)}{e_s(T)}$$

or

$$f(T,P) \times e_s(T) = e'_s(T,P)$$

As with Enhanced Saturation Vapor Pressure and Saturation Vapor Pressure, there are really two functions, one for equilibrium over water, the other for equilibrium over ice.

$$f_w(T,P) \quad \text{and} \quad f_i(T,P)$$

Dew Point

The Dew Point of a moist air sample is the temperature to which the sample must be cooled to reach saturation with respect to liquid water. Using the Enhanced Saturation Vapor Pressure function,

given

P = total pressure of moist air sample

e = partial pressure of water vapor in the sample

solve

$$e = e'_{ws}(T_d, P)$$

for

T_d = dew point temperature

The Enhanced Saturation Vapor Pressure at the Dew Point temperature and pressure P is the same as the Partial Pressure of the water vapor in the moist air sample at the current temperature T and the same pressure P .

Frost Point

The Frost Point of a moist air sample is the temperature to which the sample must be cooled to reach saturation with respect to ice. Using the Enhanced Saturation Vapor Pressure function,

given

P = total pressure of moist air sample

e = partial pressure of water vapor in the sample

solve

$$e = e'_{is}(T_f, P)$$

for

T_f = frost point temperature

The Enhanced Saturation Vapor Pressure at the Frost Point temperature and pressure P is the same as the Partial Pressure of the water vapor in the moist air sample at the current temperature T and the same pressure P .

Mixing Ratio by Volume

The Mixing Ratio by Volume of a moist air sample is really the Mixing Ratio by Moles.

mixing ratio by volume

$$\begin{aligned} &= \frac{\text{moles of water vapor}}{\text{moles of dry air}} \\ &= \frac{\text{partial pressure of water vapor}}{\text{partial pressure of dry air}} \\ &= \frac{e}{P - e} \end{aligned}$$

Here P is the pressure of the moist air sample and e is the Partial Pressure of the water vapor present in the sample. The Mixing Ratio by Volume of a moist air sample does not change when either temperature or pressure changes. The Mixing Ratio by Volume is related to the Mixing Ratio by Weight through the Molecular Weights of water vapor and dry air.

$$\text{mixing ratio by weight} = \frac{M_v}{M_a} \times \text{mixing ratio by volume}$$

Dry Air Mole Fraction

The Dry Air Mole Fraction Wexler⁽¹⁾ in a moist air sample is

$$\begin{aligned} \text{dry air mole fraction} &= \frac{\text{moles of dry air}}{\text{moles of moist air}} \\ &= \frac{\text{partial pressure of dry air}}{\text{total pressure of moist air}} \\ &= \frac{P - e}{P} \end{aligned}$$

Here P is the pressure of the moist air sample and e is the Partial Pressure of the water vapor present in the sample. The Dry Air Mole Fraction of a moist air sample does not change when either temperature or pressure changes. Dry Air Mole Fraction and Vapor Mole Fraction are related by

$$\text{dry air mole fraction} + \text{vapor mole fraction} = 1$$

Vapor Mole Fraction

The Vapor Mole Fraction Wexler⁽¹⁾ in a moist air sample is

$$\begin{aligned} \text{vapor mole fraction} &= \frac{\text{moles of water vapor}}{\text{moles of moist air}} \\ &= \frac{\text{partial pressure of water vapor}}{\text{total pressure of moist air}} \\ &= \frac{e}{P} \end{aligned}$$

Here P is the pressure of the moist air sample and e is the Partial Pressure of the water vapor present in the sample. The Vapor Mole Fraction of a moist air sample does not change when either temperature or pressure changes. Vapor Mole Fraction and Dry Air Mole Fraction are related by

$$\text{dry air mole fraction} + \text{vapor mole fraction} = 1$$

Percent by Volume

Percent by Volume Wexler⁽¹⁾ is really Percent by Moles. It is the Vapor Mole Fraction expressed as a percent. In a moist air sample,

$$\begin{aligned} \text{percent by volume} &= \frac{\text{moles of water vapor}}{\text{moles of moist air}} \times 100 \\ &= \frac{\text{partial pressure of water vapor}}{\text{total pressure of moist air}} \times 100 \\ &= \frac{e}{P} \times 100 \\ &= \text{vapor mole fraction} \times 100 \end{aligned}$$

Here P is the pressure of the moist air sample and e is the Partial Pressure of the water vapor present in the sample. The Percent by Volume of a moist air sample does not change when either temperature or pressure changes. The Percent by Volume is related to the Percent by Weight through the Molecular Weights of water vapor and dry air.

$$\text{percent by volume} = \frac{M_v}{M_a} \times \text{percent by weight}$$

Parts Per Million by Volume

Parts Per Million by Volume is really Parts Per Million by Moles. It is the Mixing Ratio by Volume expressed in parts per million. In a moist air sample,

$$\begin{aligned} \text{parts per million by volume} &= \frac{\text{moles of water vapor}}{\text{moles of dry air}} \times 10^6 \\ &= \frac{\text{partial pressure of water vapor}}{\text{partial pressure of dry air}} \times 10^6 \\ &= \frac{e}{P - e} \times 10^6 \\ &= \text{mixing ratio by volume} \times 10^6 \end{aligned}$$

Here P is the pressure of the moist air sample and e is the Partial Pressure of the water vapor present in the sample. The Parts Per Million by Volume of a moist air sample does not change when either temperature or pressure changes. The Parts per Million by Volume is related to the Parts per Million by Weight through the Molecular Weights of water vapor and dry air.

$$\text{parts per million by volume} = \frac{M_a}{M_v} \times \text{parts per million by weight}$$

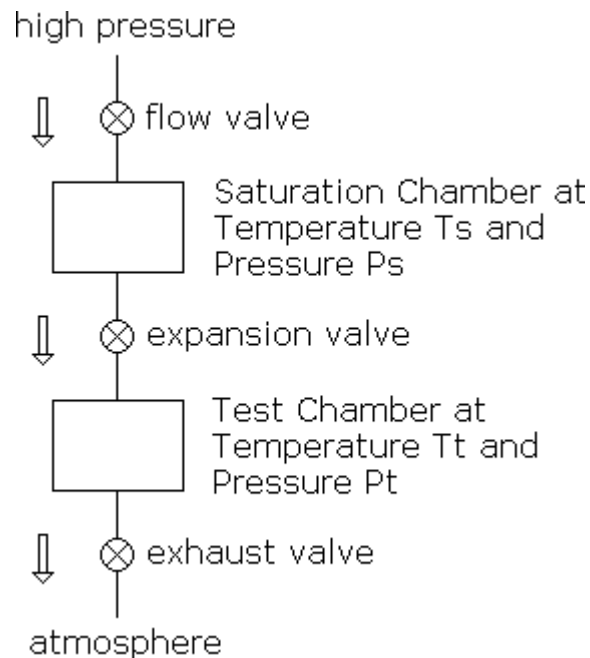
Relative Humidity

Relative Humidity is the ratio of the amount of water vapor in a sample to the maximum amount possible at the same temperature and pressure. It is expressed as a percent. In a sample of moist air at pressure P and temperature T, using the Enhanced Saturation Vapor Pressure,

$$\begin{aligned} \text{relative humidity} &= \frac{\text{partial pressure of water vapor}}{\{\text{enhanced saturation vapor pressure}\}} \times 100 \\ &= \frac{e}{e'_s(T,P)} \times 100 \end{aligned}$$

Two-Pressure/Two-Temperature Humidity Generator

A Two-Pressure/Two-Temperature Humidity Generator has a chamber, the Test Chamber, in which the humidity can be set to a predetermined value.



The pressures and temperatures in both chambers can be controlled. If the valve is not present at the Test Chamber exhaust, then Test Pressure is atmospheric pressure. If the apparatus is such that both temperatures are always the same, it is called a Two-Pressure Generator. If the apparatus is such that both pressures are always the same, it is called a Two-Temperature

Generator. The air in the Saturation Chamber is saturated. That is, Partial Pressure of the water vapor in the Saturation Chamber is

$$e'_s(T_s, P_s)$$

The same moist air flows through both chambers, so the Vapor Mole Fractions must be the same.

$$\frac{e'_s(T_s, P_s)}{P_s} = \frac{e}{P_t}$$

Where e is the Partial Pressure of the water vapor in the Test Chamber. If it is desired to achieve a specified Relative Humidity in the Test Chamber at a given test temperature and pressure, then the saturation temperature and pressure can be adjusted accordingly. Using the equation for Relative Humidity to get rid of the variable e gives

$$e'_s(T_s, P_s) \times \frac{P_t}{P_s} = \frac{\%RH}{100} \times e'_s(T_t, P_t)$$

For a desired Relative Humidity, test temperature, and test pressure, suitable values of saturation pressure and temperature must be found to satisfy this equation.

References

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Greenspan, L. "Functional Equations for the Enhancement Factors for CO₂-Free Moist Air", Journal of Research of the National Bureau of Standards - A. Physics and Chemistry, Vol. 80A, No. 1, January - February 1976; page 41, equation 3; page 42, equations 5 and 6, Table 1.