Uncertainty Analysis
of the
Thunder Scientific Model 2900 Two-Pressure Humidity Generator

By Michael Hamilton

1.0 Introduction

Described here is the generated humidity uncertainty analysis, following the Guidelines of NIST and NCSL International [1, 6, 7], for a Model 2900 Humidity Generator that utilizes the NIST developed and proven two-pressure humidity generation principle [2, 3]. Generation of humidity in a system of this type does not require direct measurements of the water vapor content of the gas. Rather, the generated humidity is derived from the measurements of saturation and chamber pressures, and saturation and chamber temperatures.

The measurement instrumentation used in both our in-house working standards and our manufactured devices are obtained from companies which have demonstrated either NIST traceability or traceability to other acceptable standards. In most cases, we use the specifications supplied by these manufacturers as the starting point for our uncertainty statements. Over time, comparison calibrations against a NIST traceable pressure gauge and NIST traceable standard resistance thermometer, as well as the results of an on-going intercomparison program of both the individual components and of the outputs of generators as a system, have allowed the determination of the ranges of disagreement among the various temperatures and pressures that go into the final determination of the output uncertainties. The average values of these disagreements represent the uncertainties from our in-house processes and things like instrument drift over time; and these are coupled with the uncertainties given by the various instrument manufacturers to give overall uncertainty statements.

This document lists the various uncertainty sources, their magnitudes, and their origins over the operating range of the Model 2900 generator (refer to the specifications section in the Model 2900 system manual for exact operating range).

2.0 Defining Equations

NIST Technical Note 1297[1] states that the uncertainty in a dependent variable, which depends only on uncorrelated input variables, is

$$u^2(y) = \sum_i u^2(x_i)\left(\frac{\partial y}{\partial x_i}\right)^2 \qquad (1)$$

Relative Humidity is defined as the amount of water vapor in a sample compared to the maximum amount possible at the given sample's temperature and pressure.

This can be expressed by the following formula:

$$\%RH = \frac{e(T_D)\cdot f(T_D, P_C)}{e(T_C)\cdot f(T_C, P_C)}\cdot \eta_S \qquad (2)$$

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Where the *f* functions are enhancement factors, *e* is the saturation vapor pressure, η_S is the % efficiency of saturation, T_C is the chamber temperature, T_D is Dew/Frost point temperature, and Pc is the chamber pressure.

The Dew/Frost point temperatures can be expressed by the following formulas:

$$e_W(T_D) \cdot f(T_D, P_C) = f(T_S, P_S) \cdot e(T_S) \cdot \frac{P_C}{P_S} \tag{3}$$

$$e_I(T_F) \cdot f(T_F, P_C) = f(T_S, P_S) \cdot e(T_S) \cdot \frac{P_C}{P_S} \tag{4}$$

Where the *f* functions are enhancement factors, e_w is the saturation vapor pressure over water, e_I is the saturation vapor pressure over ice, T_D, T_F, T_S are the Dew point, Frost point and saturation temperatures, and Pc and Ps are the chamber and saturation pressures. Note that the actual Dew/Frost point temperature is defined implicitly and must be obtained through iterative solving.

Combining equation 2 with equations 3 and 4 we can express Relative Humidity in the terms of saturation and chamber temperatures and saturation and chamber pressure only by the following formula:

$$\%RH = \frac{e(T_S) \cdot f(T_S, P_S)}{e(T_C) \cdot f(T_C, P_C)} \cdot \frac{P_C}{P_S} \cdot \eta_S \tag{5}$$

By incorporating the relationship in equation 1 into an uncertainty equation of the form of equation 5, it can be shown that the total uncertainty in relative humidity is given by the expression:

$$u^2(RH) = u^2(T_C)\left(\frac{\partial RH}{\partial T_C}\right)^2 + u^2(T_S)\left(\frac{\partial RH}{\partial T_S}\right)^2 + u^2(P_C)\left(\frac{\partial RH}{\partial P_C}\right)^2 + u^2(P_S)\left(\frac{\partial RH}{\partial P_S}\right)^2 + u^2(\eta_S)\left(\frac{\partial RH}{\partial \eta_S}\right)^2 \tag{6}$$

Similarly incorporating the relationship in equation 1 into an uncertainty equation of the form of equation 3 and 4, the uncertainties in dew point and frost point measurement are given by the expression:

$$u^2(T_D) = u^2(T_S)\left(\frac{\partial T_D}{\partial T_S}\right)^2 + u^2(P_C)\left(\frac{\partial T_D}{\partial P_C}\right)^2 + u^2(P_S)\left(\frac{\partial T_D}{\partial P_S}\right)^2 + u^2(\eta_S)\left(\frac{\partial T_D}{\partial \eta_S}\right)^2 \tag{7}$$

and

$$u^2(T_F) = u^2(T_S)\left(\frac{\partial T_F}{\partial T_S}\right)^2 + u^2(P_C)\left(\frac{\partial T_F}{\partial P_C}\right)^2 + u^2(P_S)\left(\frac{\partial T_F}{\partial P_S}\right)^2 + u^2(\eta_S)\left(\frac{\partial T_F}{\partial \eta_S}\right)^2 \tag{8}$$

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3 Uncertainty Components

In the mathematical analysis of equations 6, 7 and 8, we will analyze the uncertainties due to each component separately and then combine the uncertainties to obtain the total expanded uncertainty. We are concerned with four basic categories of uncertainty; pressure, temperature, the saturation vapor pressure/enhancement factor equations and percent efficiency of the saturator. Each of these categories may also have associated uncertainty components. In determining components of uncertainty, there are several things to consider, such as measurement accuracy or uncertainty, measurement resolution, uniformity, etc.

Listed below are the identified major uncertainty contributors and their components for the Model 2900 humidity generator.

- Uncertainty contribution from pressure (P_s and P_c) which includes
  - Measurement accuracy
    - Reference standard
    - Linearity
    - Drift
    - Hysteresis
    - Temperature effects over the calibrated range
    - Repeatability
  - Measurement resolution

- Uncertainty contribution from temperature (T_s and T_c), which includes
  - Measurement accuracy
    - Reference standard
    - Measurement resolution
    - Module error
    - Hysteresis
    - Self-Heating
    - Control Stability (Repeatability)
  - Chamber Uniformity (Applies to Chamber Temperature (T_c) only)

- Uncertainty contribution from Equations, which includes
  - Saturation Vapor Pressure Equation (e(T))
  - Enhancement Factor Equation (f(T,P))

- Uncertainty contribution from percent efficiency of the saturator (η_S)

3.1 Pressure Uncertainty Contribution

The pressure terms, P_c or P_s, in a two-pressure humidity generator are major determining factors. The Model 2900 humidity generator uses one pressure transducer to measure the saturation pressure and another pressure transducer to measure the chamber pressure. In this design, each pressure transducer contributes its own uncertainty to the overall system and will be addressed independent of one another.

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The pressure uncertainty contribution in terms of relative humidity can be determined by the partial numeric differential of the RH equation with respect to pressure, multiplied by the uncertainty of the pressure component. The equation for this becomes.

$$\mathrm{uRH}_{[\mathrm{component}]} = \frac{\partial}{\partial P}\left[\frac{e_S(T_S)\cdot f(T_S,P_S)}{e_S(T_C)\cdot f(T_C,P_C)}\cdot\frac{P_C}{P_S}\cdot\eta_S\right]\cdot \mathrm{uP}_{[\mathrm{component}]} \tag{9}$$

$\mathrm{uRH}_{[\mathrm{component}]}$ = Pressure component uncertainty in terms of percent relative humidity.

$\mathrm{uP}_{[\mathrm{component}]}$ = Pressure component uncertainty in terms of pressure.

The pressure uncertainty contribution in terms of dew or frost point temperature can be determined by the partial numeric differential of the iterative dew or frost point equation with respect to pressure, multiplied by the uncertainty of the pressure component. The equation for this becomes

$$\mathrm{uT}_{\mathrm{D}[\mathrm{component}]} = \frac{\partial}{\partial P}\left[e_W(T_D)\cdot f(T_D,P) = f(T_S,P_S)\cdot e(T_S)\cdot\frac{P_C}{P_S}\right]\cdot \mathrm{uP}_{[\mathrm{component}]} \tag{10}$$

$$\mathrm{uT}_{\mathrm{F}[\mathrm{component}]} = \frac{\partial}{\partial P}\left[e_I(T_F)\cdot f(T_F,P) = f(T_S,P_S)\cdot e(T_S)\cdot\frac{P_C}{P_S}\right]\cdot \mathrm{uP}_{[\mathrm{component}]} \tag{11}$$

$\mathrm{uT}_{\mathrm{D}[\mathrm{component}]}$ = Pressure component uncertainty in terms of dew point temperature.

$\mathrm{uT}_{\mathrm{F}[\mathrm{component}]}$ = Pressure component uncertainty in terms of frost point temperature.

$\mathrm{uP}_{[\mathrm{component}]}$ = Pressure component uncertainty in terms of pressure.

3.1.1 Pressure Accuracy Uncertainty Component

Pressure measurement accuracy of Model 2900 humidity generator's saturation pressure transducer is specified by the manufacturer as 0.02% of the full scale. This total manufacturer uncertainty (k=2) includes reference standard, linearity, drift, hysteresis, temperature effects over the calibrated range and repeatability. Taking a conservative approach that is based on a rectangular distribution, the uncertainty component of the saturation pressure accuracy is then

$\mathrm{uP}_{\mathrm{s}[\mathrm{accuracy}]}$ = (160 psia (full scale) * 0.02%) / √3
= (0.032 psia) / √3 (DOF=infinite)

Pressure measurement accuracy of Model 2900 humidity generator's chamber pressure transducer is specified by the manufacturer as 0.02% of reading. This total manufacturer uncertainty (k=2) includes reference standard, linearity, drift, hysteresis, temperature effects over the calibrated range and repeatability. Taking a conservative approach that is based on a rectangular distribution, the uncertainty component of the chamber pressure accuracy is then

$\mathrm{uP}_{\mathrm{c}[\mathrm{accuracy}]}$ = (14.7 psia (of reading) * 0.02%) / √3
= (0.00294 psia) / √3 (DOF=infinite)

Note: This analysis will use standard pressure as the chamber pressure reading for all calculations

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3.1.2 Pressure Resolution Uncertainty Component

The Model 2900 humidity generator digitally communicates with both the saturation and chamber pressure transducers. Based on a rectangular distribution of the half-interval of resolution, the uncertainty component of pressure resolution is then

$$uP_{s\,[resolution]} = 0.001 \text{ psia} / \sqrt{12} \text{ (DOF=infinite)}$$

$$uP_{c\,[resolution]} = 0.001 \text{ psia} / \sqrt{12} \text{ (DOF=infinite)}$$

3.1.3 Pressure Uncertainty Contribution Summary

The standard %RH uncertainties, uRH, components calculated using equation 9 from the associated individual pressure components are summarized in Table 1 and Figure 1.

Standard Pressure Uncertainty Components of RH (%)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		98.0%RH	**73.6%RH**	**49.2%RH**	**37.0%RH**	**29.6%RH**	**19.9%RH**	**15.0%RH**	**10.1%RH**		
0 °C	Ps Accuracy	0.120280	0.067657	0.030069	0.016914	0.010824	0.004810	0.002706	0.001202	Infinity	Type B
	Ps Resolution	0.001879	0.001057	0.000470	0.000264	0.000169	0.000075	0.000042	0.000019	Infinity	Type B
	Pc Accuracy	0.011277	0.008468	0.005659	0.004254	0.003412	0.002288	0.001726	0.001165	Infinity	Type B
	Pc Resolution	0.000019	0.000014	0.000010	0.000007	0.000006	0.000004	0.000003	0.000002	Infinity	Type B
	Combined	*0.120822*	*0.068193*	*0.030601*	*0.017443*	*0.011351*	*0.005327*	*0.003210*	*0.001674*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.1%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
35 °C	Ps Accuracy	0.120373	0.067713	0.030096	0.016929	0.010835	0.004815	0.002708	0.001204	Infinity	Type B
	Ps Resolution	0.001881	0.001058	0.000470	0.000265	0.000169	0.000075	0.000042	0.000019	Infinity	Type B
	Pc Accuracy	0.011286	0.008472	0.005658	0.004251	0.003407	0.002282	0.001719	0.001156	Infinity	Type B
	Pc Resolution	0.000019	0.000014	0.000010	0.000007	0.000006	0.000004	0.000003	0.000002	Infinity	Type B
	Combined	*0.120915*	*0.068249*	*0.030627*	*0.017457*	*0.011359*	*0.005329*	*0.003208*	*0.001669*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.2%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
70 °C	Ps Accuracy	0.120218	0.067690	0.030114	0.016948	0.010850	0.004824	0.002714	0.001206	Infinity	Type B
	Ps Resolution	0.001878	0.001058	0.000471	0.000265	0.000170	0.000075	0.000042	0.000019	Infinity	Type B
	Pc Accuracy	0.011270	0.008463	0.005653	0.004247	0.003403	0.002277	0.001714	0.001151	Infinity	Type B
	Pc Resolution	0.000019	0.000014	0.000010	0.000007	0.000006	0.000004	0.000003	0.000002	Infinity	Type B
	Combined	*0.120759*	*0.068225*	*0.030644*	*0.017474*	*0.011372*	*0.005335*	*0.003210*	*0.001667*	*Infinity*	

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Table 1

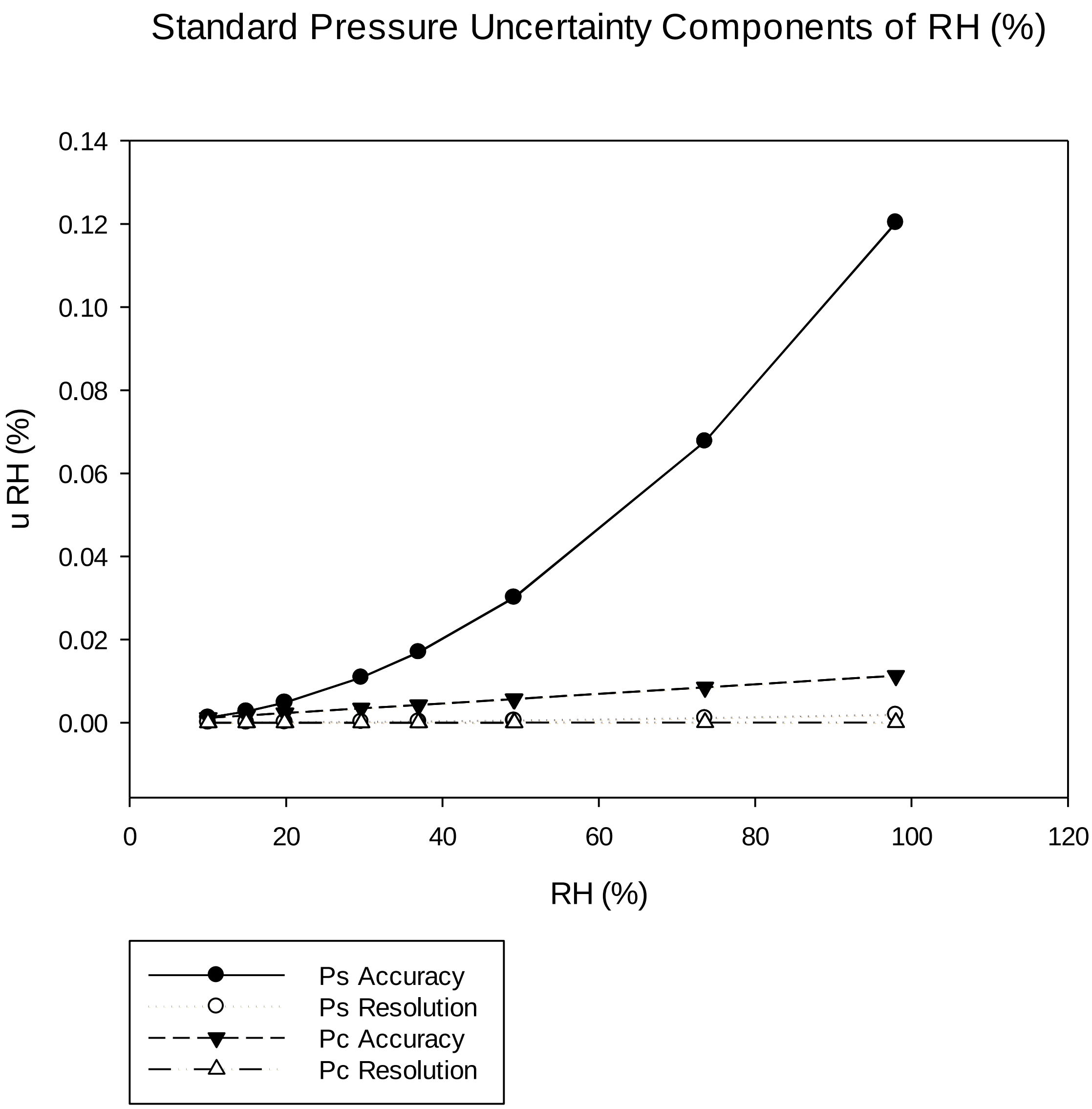

Figure 1

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The standard Dew Point Temperature uncertainties, uTD, components calculated using equation 10 from the associated individual pressure components are summarized in Table 2 and Figure 2.

Standard Pressure Uncertainty Components of Dew Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.3 °C	**-4.1 °C**	**-9.4 °C**	**-13.0 °C**	**-15.7 °C**	**-20.4 °C**	**-23.6 °C**	**-27.9 °C**		
0 °C	Ps Accuracy	0.016854	0.012227	0.007782	0.005648	0.004404	0.002801	0.002029	0.001285	Infinity	Type B
	Ps Resolution	0.000263	0.000191	0.000122	0.000088	0.000069	0.000044	0.000032	0.000020	Infinity	Type B
	Pc Accuracy	0.001580	0.001530	0.001464	0.001420	0.001387	0.001331	0.001294	0.001244	Infinity	Type B
	Pc Resolution	0.000003	0.000003	0.000002	0.000002	0.000002	0.000002	0.000002	0.000002	Infinity	Type B
	Combined	*0.016930*	*0.012324*	*0.007919*	*0.005824*	*0.004618*	*0.003101*	*0.002407*	*0.001789*	*Infinity*	
		34.6 °C	**29.6 °C**	**22.7 °C**	**18.1 °C**	**14.6 °C**	**8.6 °C**	**4.5 °C**	**-1.1 °C**		
35 °C	Ps Accuracy	0.022130	0.015971	0.010094	0.007292	0.005668	0.003585	0.002590	0.001635	Infinity	Type B
	Ps Resolution	0.000346	0.000250	0.000158	0.000114	0.000089	0.000056	0.000040	0.000026	Infinity	Type B
	Pc Accuracy	0.002075	0.001998	0.001897	0.001831	0.001782	0.001698	0.001643	0.001570	Infinity	Type B
	Pc Resolution	0.000004	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	0.000003	Infinity	Type B
	Combined	*0.022230*	*0.016097*	*0.010272*	*0.007519*	*0.005942*	*0.003968*	*0.003067*	*0.002267*	*Infinity*	
		69.5 °C	**63.1 °C**	**54.4 °C**	**48.6 °C**	**44.3 °C**	**36.7 °C**	**31.6 °C**	**24.8 °C**		
70 °C	Ps Accuracy	0.028269	0.020280	0.012723	0.009149	0.007086	0.004457	0.003208	0.002017	Infinity	Type B
	Ps Resolution	0.000442	0.000317	0.000199	0.000143	0.000111	0.000070	0.000050	0.000032	Infinity	Type B
	Pc Accuracy	0.002650	0.002537	0.002391	0.002295	0.002225	0.002107	0.002029	0.001927	Infinity	Type B
	Pc Resolution	0.000005	0.000004	0.000004	0.000004	0.000004	0.000004	0.000003	0.000003	Infinity	Type B
	Combined	*0.028396*	*0.020441*	*0.012947*	*0.009433*	*0.007428*	*0.004931*	*0.003796*	*0.002790*	*Infinity*	

Table 2

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Standard Pressure Uncertainty Components of Dew Point Temperature (°C)

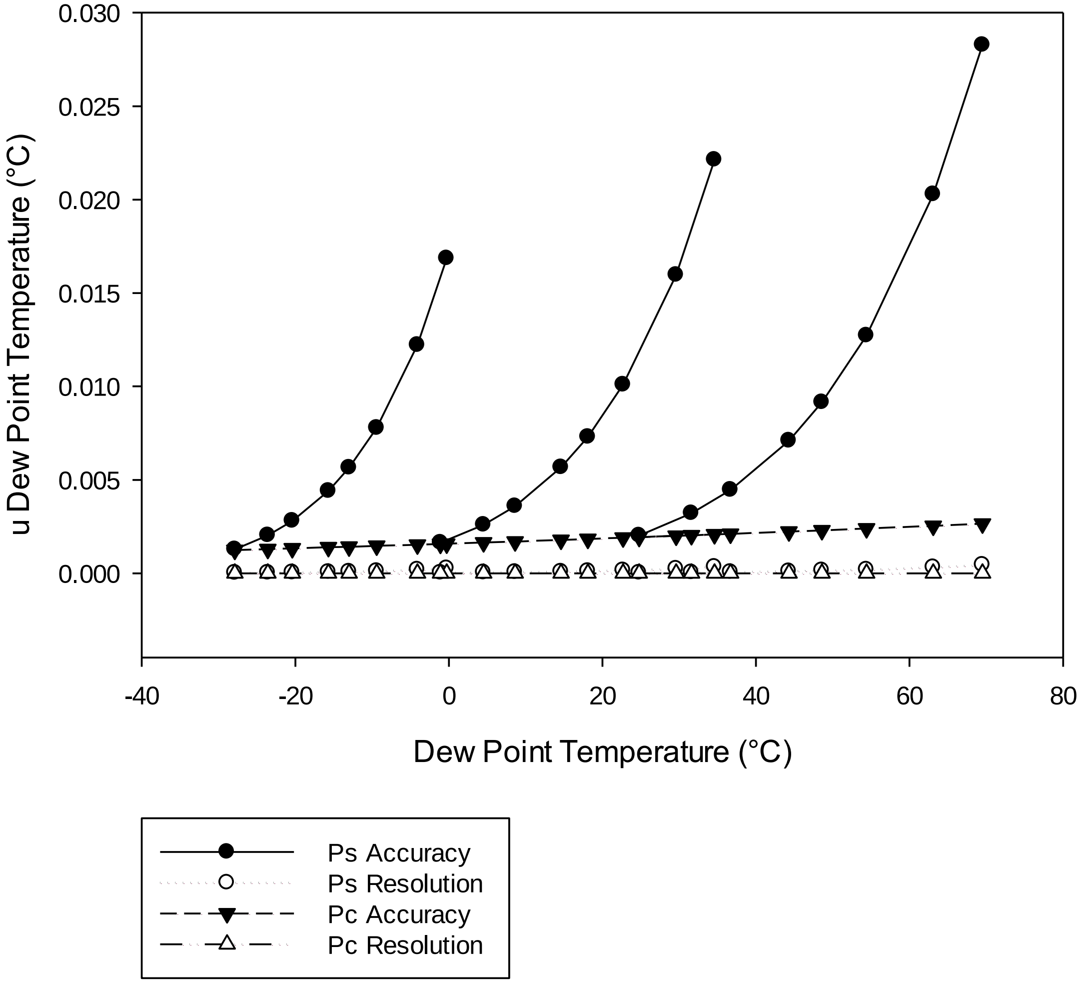

Figure 2

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The standard Frost Point Temperature uncertainties, uT_F, components calculated using equation 11 from the associated individual pressure components are summarized in Table 3 and Figure 3.

Note: Any frost point value that is not possible is grayed out of the following table.

Standard Pressure Uncertainty Components of Frost Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.2 °C	**-3.7 °C**	**-8.4 °C**	**-11.6 °C**	**-14.0 °C**	**-18.3 °C**	**-21.2 °C**	**-25.2 °C**		
0 °C	Ps Accuracy	0.014875	0.010863	0.006974	0.005092	0.003988	0.002555	0.001861	0.001187	Infinity	Type B
	Ps Resolution	0.000232	0.000170	0.000109	0.000080	0.000062	0.000040	0.000029	0.000019	Infinity	Type B
	Pc Accuracy	0.001395	0.001359	0.001312	0.001280	0.001256	0.001215	0.001186	0.001149	Infinity	Type B
	Pc Resolution	0.000002	0.000002	0.000002	0.000002	0.000002	0.000002	0.000002	0.000002	Infinity	Type B
	Combined	*0.014942*	*0.010949*	*0.007097*	*0.005251*	*0.004181*	*0.002830*	*0.002207*	*0.001652*	*Infinity*	
									-0.9 °C		
35 °C	Ps Accuracy								0.001445	Infinity	Type B
	Ps Resolution								0.000023	Infinity	Type B
	Pc Accuracy								0.001387	Infinity	Type B
	Pc Resolution								0.000002	Infinity	Type B
	Combined								*0.002003*	*Infinity*	
70 °C	Ps Accuracy									Infinity	Type B
	Ps Resolution									Infinity	Type B
	Pc Accuracy									Infinity	Type B
	Pc Resolution									Infinity	Type B
	Combined									*Infinity*	

Table 3

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Standard Pressure Uncertainty Components of Frost Point Temperature (°C)

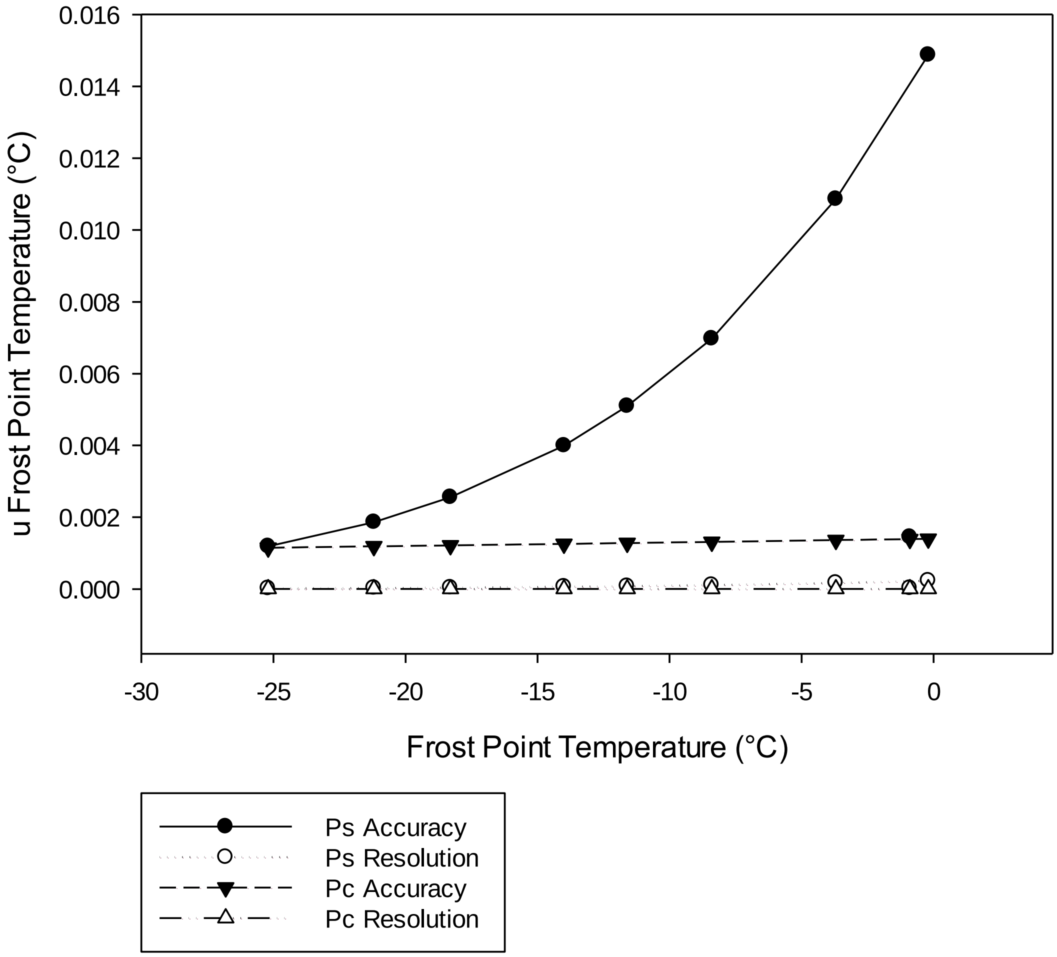

Figure 3

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3.2 Temperature Uncertainty Contribution

The temperature terms, T_c or T_s, in a two-pressure humidity generator are another major contributor of uncertainty and are used mathematically to calculate saturation vapor pressures. The Model 2900 humidity generator uses one temperature probe to measure the saturation temperature and another temperature probe to measure the chamber temperature. In this design, each temperature probe contributes its own uncertainty to the overall system and will be addressed independent of one another.

3.2.1 Saturation Temperature Uncertainty Contribution

The saturation temperature uncertainty contribution in terms of relative humidity can be determined by the partial numeric differential of the RH equation with respect to saturation temperature, multiplied by the uncertainty of the saturation temperature component. The equation for this becomes

$$\mathrm{uRH}_{[\text{component}]} = \frac{\partial}{\partial T_S}\left[\frac{e_S(T_S)\cdot f(T_S,P_S))}{e_S(T_C)\cdot f(T_C,P_C)}\cdot\frac{P_C}{P_S}\cdot\eta_S\right]\cdot \mathrm{u}T_{S[\text{component}]} \tag{15}$$

$\mathrm{uRH}_{[\text{component}]}$ = Saturation Temperature component uncertainty in terms of percent relative humidity.

$\mathrm{uT}_{S[\text{component}]}$ = Saturation Temperature component uncertainty in terms of temperature.

The saturation temperature uncertainty contribution in terms of dew or frost point temperature can be determined by the partial numeric differential of the iterative dew or frost point equation with respect to saturation temperature, multiplied by the uncertainty of the saturation temperature component. The equations for these become

$$\mathrm{uT}_{D[\text{component}]} = \frac{\partial}{\partial T_S}\left[e_W(T_D)\cdot f(T_D,P_C) = f(T_S,P_S)\cdot e(T_S)\cdot\frac{P_C}{P_S}\right]\cdot \mathrm{uT}_{S\,[\text{component}]} \tag{16}$$

$$\mathrm{uT}_{F[\text{component}]} = \frac{\partial}{\partial T_S}\left[e_I(T_F)\cdot f(T_F,P_C) = f(T_S,P_S)\cdot e(T_S)\cdot\frac{P_C}{P_S}\right]\cdot \mathrm{uT}_{S\,[\text{component}]} \tag{17}$$

$\mathrm{uT}_{D[\text{component}]}$ = Saturation Temperature component uncertainty in terms of dew point temperature.

$\mathrm{uT}_{F[\text{component}]}$ = Saturation Temperature component uncertainty in terms of frost point temperature

$\mathrm{uT}_{S[\text{component}]}$ = Saturation Temperature component uncertainty in terms of temperature.

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3.2.1.1 Saturation Temperature Measurement Uncertainty Component

Temperature measurement accuracy of Model 2900 humidity generator's saturation temperature probe encompasses 7 separate components that are listed and combined in Table 4.

Model 2900 Temperature Components of Uncertainty					
Description	Uncertainty (±)	k=	Distribution	Degrees of Freedom	Evaluation
Standard	0.02	2	Normal	Infinity	Type B
Resolution	0.0000625849	1	Resolution	Infinity	Type B
Offset Error	0.00237	1	Rectangular	Infinity	Type B
Gain Error	0.00002765	1	Rectangular	Infinity	Type B
Hysteresis	0.015	1	Rectangular	Infinity	Type B
Self-Heating	0.003	1	Rectangular	Infinity	Type B
Fluid Control Stability	0.002	1	Normal	Infinity	Type A
				Combined Standard Uncertainty (±):	0.01356
				Effective Degrees of Freedom:	Infinity
				Confidence:	95.45%
				k:	2
				Expanded Combined Uncertainty (±):	0.02712

Table 4

Using the expanded result from table 4 and taking a conservative approach that is based on a rectangular distribution, the uncertainty component of saturation temperature accuracy is then

$$uT_{s[accuracy]} = 0.027\ °C / \sqrt{3}\ (DOF=infinite)$$

resulting in

$$uT_{s[accuracy]} = 0.031\ °C$$

(using a coverage factor, k=2, at an approximate level of confidence of 95%)

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3.2.2 Chamber Temperature Uncertainty Contribution

The chamber temperature uncertainty contribution in terms of relative humidity can be determined by the partial numeric differential of the RH equation with respect to chamber temperature, multiplied by the uncertainty of the chamber temperature component. The equation for this becomes

$$\mathrm{uRH}_{[\mathrm{component}]} = \frac{\partial}{\partial T_C}\left[\frac{e_S(T_S)\cdot f(T_S,P_S)}{e_S(T_C)\cdot f(T_C,P_C)}\cdot\frac{P_C}{P_S}\cdot\eta_S\right]\cdot \mathrm{u}T_{C[\mathrm{component}]} \tag{18}$$

$\mathrm{uRH}_{[\mathrm{component}]}$ = Chamber Temperature component uncertainty in terms of percent relative humidity.

$\mathrm{uT}_{\mathrm{C[component]}}$ = Chamber Temperature component uncertainty in terms of temperature.

Examining equations 3 and 4, dew and frost point equations, we see that the chamber temperature has no component and therefore no uncertainty contribution to the generated dew or frost point temperatures.

3.2.2.1 Chamber Temperature Measurement Uncertainty Component

Temperature measurement accuracy of Model 2900 humidity generator's chamber temperature probe encompasses 7 separate components that are the same as those listed in Table 4. Using the expanded result from table 4 and taking a conservative approach that is based on a rectangular distribution, the uncertainty component of chamber temperature accuracy is then

$\mathrm{uT}_{\mathrm{c[accuracy]}}$ = 0.027 °C / √3 (DOF=infinite)

resulting in

$\mathrm{uT}_{\mathrm{c[accuracy]}}$ = 0.031 °C
(using a coverage factor, k=2, at an approximate level of confidence of 95%)

3.2.2.2 Chamber Temperature Uniformity Uncertainty Component

The 2900 operates by generating the %RH setpoint based on pressure and temperatures which include the chamber temperature probe. This means any temperature difference in the chamber is automatically compensated by the system based on the actual chamber temperature probe reading. In scenarios where the device under test is not bundled with the chamber temperature probe or when there are multiple devices under test in the chamber then temperature gradients within the chamber can contribute to the overall %RH uncertainty of the generator. Using the expanded result from the "Model 2900 Chamber Temperature Uniformity Analysis" [9] that is based on a rectangular distribution, the uncertainty component of chamber temperature uniformity is then

$\mathrm{uT}_{\mathrm{c[uniformity]}}$ = 0.03 °C / √3 (DOF=infinite)

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Different door configurations add a non-uniformity contributor due to the different material's thermal insulator properties. Using the expanded result from the "Model 2900 Chamber Temperature Uniformity Analysis" [9] that is based on a rectangular distribution, the uncertainty component of chamber temperature non-uniformity is then

$$uT_{c[\text{non-uniformity}]} = (0.00034 * |Tc|)\ °C / \sqrt{3}\ (\text{DOF=infinite})$$

where Tc is the chamber temperature in °C.

3.2.3 Temperature Uncertainty Contribution Summary

The standard uncertainties, uRH, components calculated using equations 15 and 18 from the associated individual temperature components previously shown are summarized in Table 5 and Figure 4.

Standard Temperature Uncertainty Components of RH (%)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		98.0%RH	**73.6%RH**	**49.2%RH**	**37.0%RH**	**29.6%RH**	**19.9%RH**	**15.0%RH**	**10.1%RH**		
0 °C	Ts Accuracy	0.110996	0.083335	0.055673	0.041843	0.033544	0.022480	0.016949	0.011418	Infinity	Type B
	Tc Accuracy	0.110997	0.083348	0.055698	0.041874	0.033579	0.022521	0.016992	0.011464	Infinity	Type B
	Tc Uniformity	0.123330	0.092608	0.061887	0.046527	0.037310	0.025023	0.018880	0.012738	Infinity	Type B
	Tc Non-Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Combined	*0.199627*	*0.149893*	*0.100159*	*0.075292*	*0.060373*	*0.040481*	*0.030536*	*0.020593*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.1%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
35 °C	Ts Accuracy	0.084562	0.063476	0.042386	0.031840	0.025512	0.017075	0.012856	0.008638	Infinity	Type B
	Tc Accuracy	0.084563	0.063481	0.042398	0.031856	0.025531	0.017097	0.012881	0.008665	Infinity	Type B
	Tc Uniformity	0.093958	0.070534	0.047109	0.035395	0.028367	0.018997	0.014312	0.009627	Infinity	Type B
	Tc Non-Uniformity	0.037270	0.027979	0.018686	0.014040	0.011252	0.007535	0.005677	0.003819	Infinity	Type B
	Combined	*0.156585*	*0.117545*	*0.078502*	*0.058979*	*0.047265*	*0.031647*	*0.023838*	*0.016030*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.2%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
70 °C	Ts Accuracy	0.066087	0.049661	0.033192	0.024941	0.019985	0.013371	0.010061	0.006750	Infinity	Type B
	Tc Accuracy	0.066083	0.049622	0.033145	0.024900	0.019951	0.013350	0.010049	0.006748	Infinity	Type B
	Tc Uniformity	0.073425	0.055135	0.036827	0.027666	0.022168	0.014834	0.011166	0.007497	Infinity	Type B
	Tc Non-Uniformity	0.058251	0.043741	0.029216	0.021949	0.017586	0.011768	0.008858	0.005948	Infinity	Type B
	Combined	*0.132359*	*0.099407*	*0.066409*	*0.049892*	*0.039977*	*0.026749*	*0.020133*	*0.013516*	*Infinity*	

Table 5

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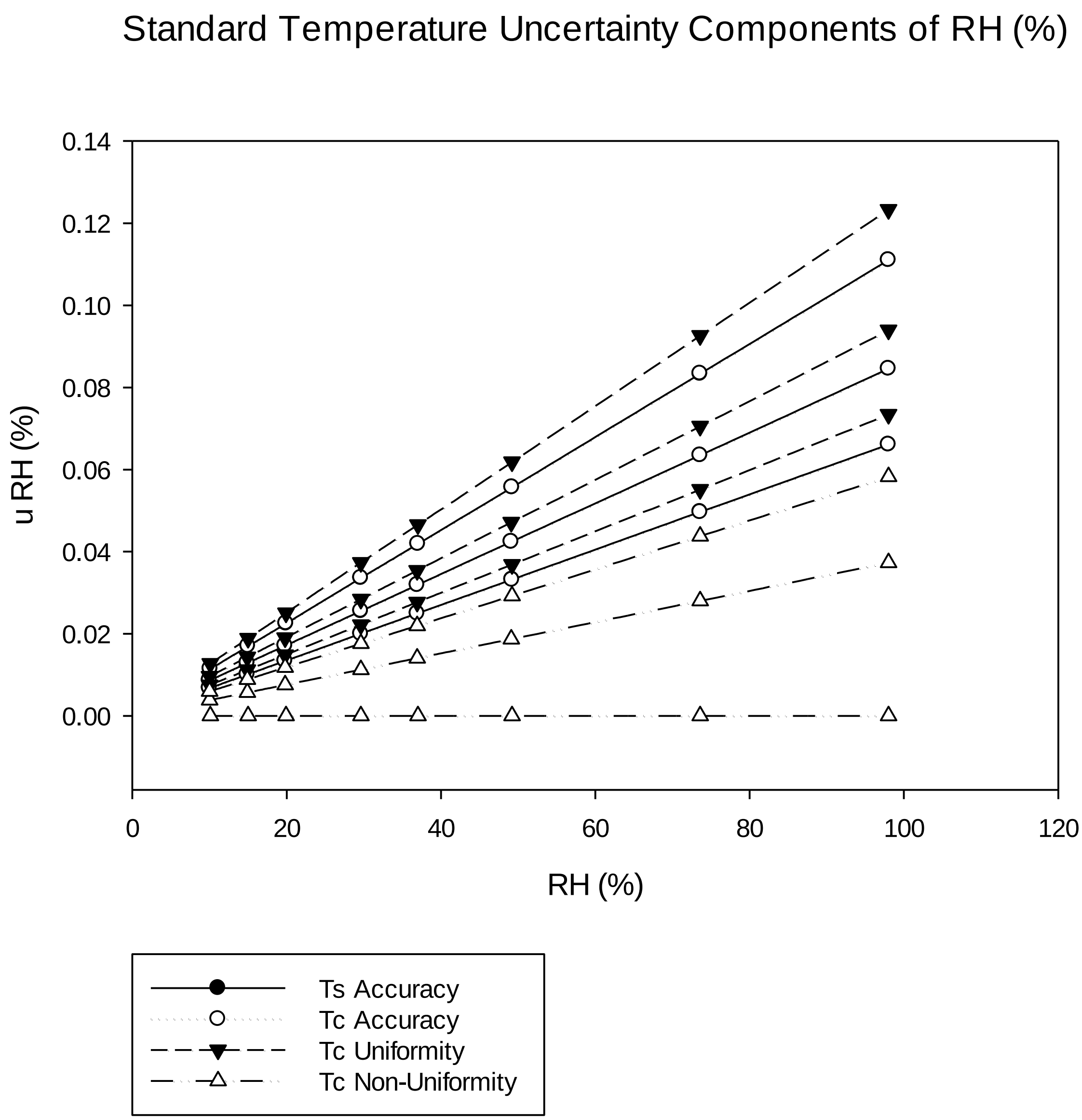

Figure 4

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The standard uncertainties, uTD, components calculated using equation 16 from the associated individual temperature components previously shown are summarized in Table 6 and Figure 5.

Standard Temperature Uncertainty Components of Dew Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.3 °C	**-4.1 °C**	**-9.4 °C**	**-13.0 °C**	**-15.7 °C**	**-20.4 °C**	**-23.6 °C**	**-27.9 °C**		
0 °C	Ts Accuracy	0.015553	0.015061	0.014408	0.013972	0.013648	0.013088	0.012712	0.012209	Infinity	Type B
	Tc Accuracy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Non-Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Combined	*0.015553*	*0.015061*	*0.014408*	*0.013972*	*0.013648*	*0.013088*	*0.012712*	*0.012209*	*Infinity*	
		34.6 °C	**29.6 °C**	**22.7 °C**	**18.1 °C**	**14.6 °C**	**8.6 °C**	**4.5 °C**	**-1.1 °C**		
35 °C	Ts Accuracy	0.015547	0.014971	0.014215	0.013715	0.013346	0.012714	0.012293	0.011736	Infinity	Type B
	Tc Accuracy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Non-Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Combined	*0.015547*	*0.014971*	*0.014215*	*0.013715*	*0.013346*	*0.012714*	*0.012293*	*0.011736*	*Infinity*	
		69.5 °C	**63.1 °C**	**54.4 °C**	**48.6 °C**	**44.3 °C**	**36.7 °C**	**31.6 °C**	**24.8 °C**		
70 °C	Ts Accuracy	0.015540	0.014879	0.014023	0.013463	0.013053	0.012355	0.011895	0.011289	Infinity	Type B
	Tc Accuracy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Non-Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Combined	*0.015540*	*0.014879*	*0.014023*	*0.013463*	*0.013053*	*0.012355*	*0.011895*	*0.011289*	*Infinity*	

Table 6

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Standard Temperature Uncertainty Components of Dew Point Temperature (°C)

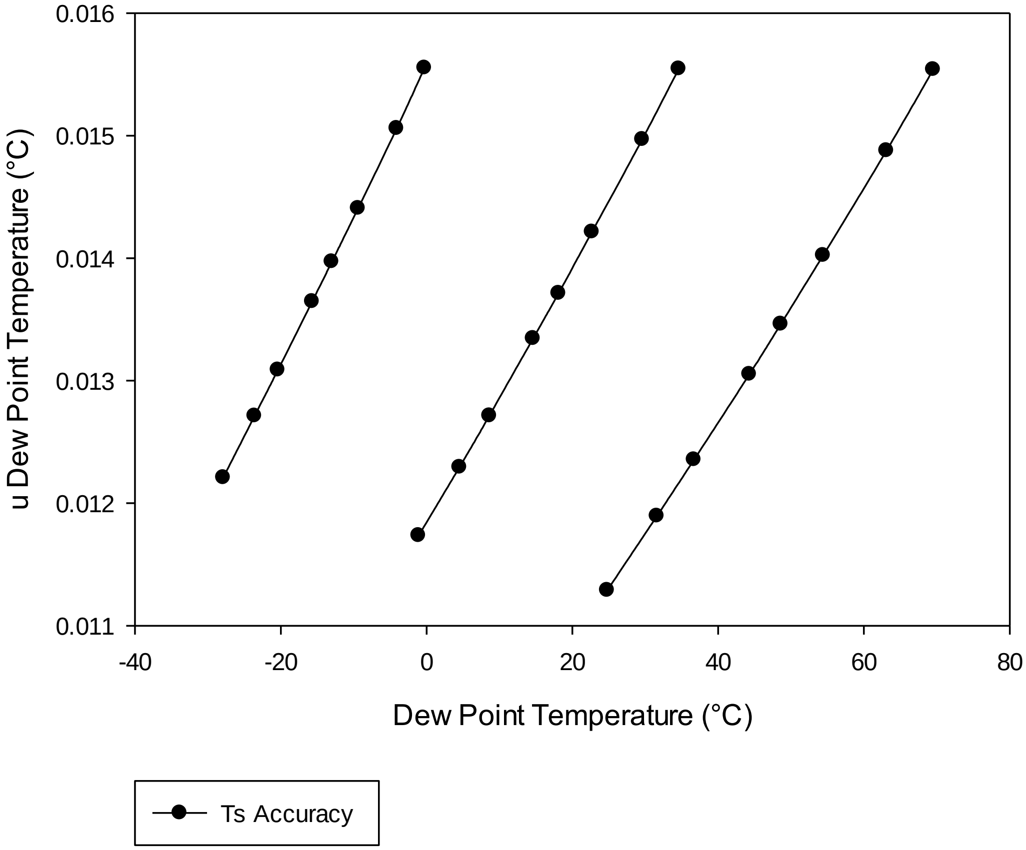

Figure 5

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The standard uncertainties, uT_F, components calculated using equation 17 from the associated individual temperature components previously shown are summarized in Table 7 and Figure 7.

Note: Any frost point value that is not possible is grayed out of the following table.

Standard Temperature Uncertainty Components of Frost Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.2 °C	**-3.7 °C**	**-8.4 °C**	**-11.6 °C**	**-14.0 °C**	**-18.3 °C**	**-21.2 °C**	**-25.2 °C**		
0 °C	Ts Accuracy	0.013730	0.013380	0.012913	0.012596	0.012358	0.011942	0.011659	0.011274	Infinity	Type B
	Tc Accuracy	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Tc Non-Uniformity	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	Infinity	Type B
	Combined	*0.013730*	*0.013380*	*0.012913*	*0.012596*	*0.012358*	*0.011942*	*0.011659*	*0.011274*	*Infinity*	
									-0.9 °C		
35 °C	Ts Accuracy								0.010372	Infinity	Type B
	Tc Accuracy								0.0	Infinity	Type B
	Tc Uniformity								0.0	Infinity	Type B
	Tc Non-Uniformity								0.0	Infinity	Type B
	Combined								*0.010372*	*Infinity*	
70 °C	Ts Accuracy									Infinity	Type B
	Tc Accuracy									Infinity	Type B
	Tc Uniformity									Infinity	Type B
	Tc Non-Uniformity									Infinity	Type B
	Combined									*Infinity*	

Table 7

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Standard Temperature Uncertainty Components of Frost Point Temperature (°C)

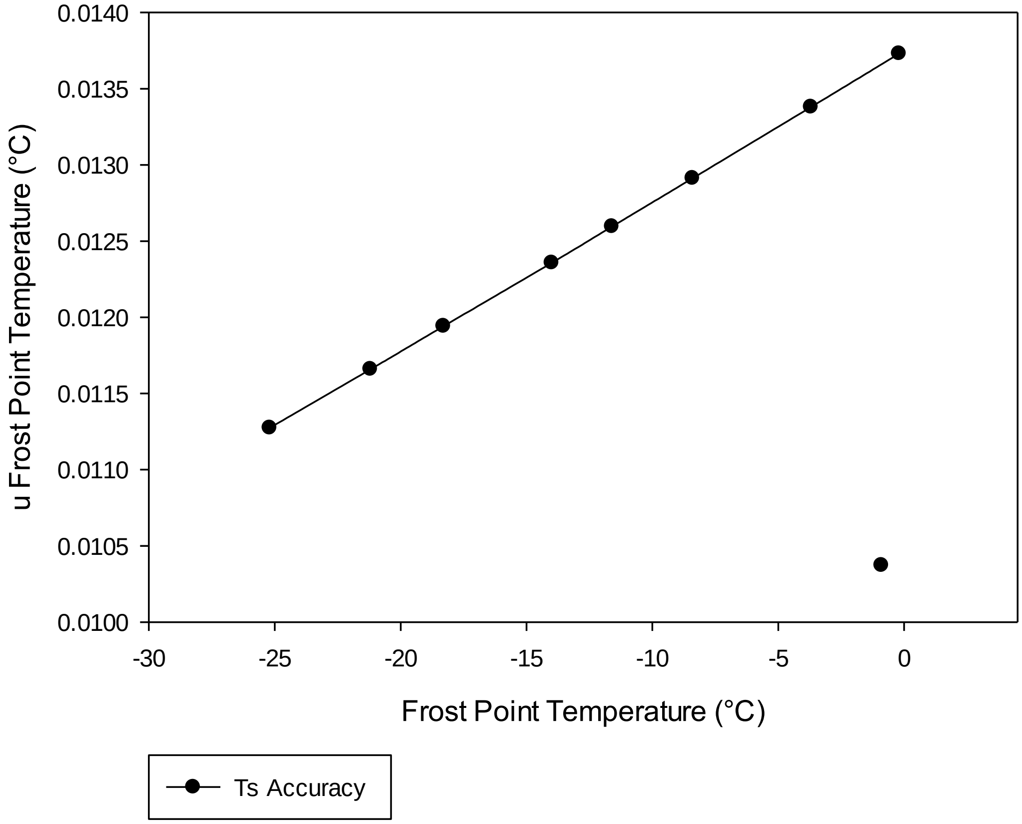

Figure 6

3.3 Equation Uncertainty Contribution

The equations used to calculate the saturation vapor pressure at a given temperature and its enhancement factor at the same temperature and given pressure have published uncertainties as determined by the author or authors of the equations. These equations are used throughout the Relative Humidity, Dew point and Frost point equations and therefore contribute their own uncertainty to the overall system.

3.3.1 Saturation Vapor Pressure Equation Uncertainty Component

The saturation vapor pressure is the partial pressure of the water vapor at a given temperature with respect to ice or water. The saturation vapor pressure is dependent on temperature only and is computed with the Wexler's [4] saturation vapor pressure equation. Wexler [4] also list a table of uncertainties at various temperatures for his saturation vapor pressure equation. These uncertainty values are interpolated to determine the saturation vapor pressure equation uncertainty component for a given temperature.

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3.3.2 Enhancement Factor Equation Uncertainty Component

Enhancement factors are slight correction factors used to account for the non-ideal behavior of water vapor when admixed with other gases. The enhancement factor is dependent on both temperature and pressure and is computed with Greenspan's [5] enhancement factor equation. Wexler and R.W. Hyland [8] list a table of uncertainties for various temperatures and pressures for the enhancement factor equation. These uncertainty values are interpolated to determine the enhancement factors equation uncertainty component for a given temperature and pressure.

3.3.3 Equation Uncertainty Contribution Summary

The standard uncertainties, uRH, components calculated using the associated equation uncertainty tables mentioned above are summarized in Table 8 and Figure 7.

Standard Equation Uncertainty Components of RH (%)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		98.0%RH	**73.6%RH**	**49.2%RH**	**37.0%RH**	**29.6%RH**	**19.9%RH**	**15.0%RH**	**10.1%RH**		
0 °C	SVP@Ts	0.001562	0.001172	0.000781	0.000586	0.000469	0.000312	0.000234	0.000156	Infinity	Type B
	F@Ts,Ps	0.010136	0.010148	0.010172	0.010197	0.010221	0.010249	0.010060	0.009967	Infinity	Type B
	SVP@Tc	0.001563	0.001173	0.000784	0.000589	0.000473	0.000317	0.000239	0.000161	Infinity	Type B
	F@Tc,Pc	0.009933	0.007459	0.004985	0.003747	0.003005	0.002015	0.001521	0.001026	Infinity	Type B
	Combined	*0.014363*	*0.012703*	*0.011382*	*0.010895*	*0.010675*	*0.010454*	*0.010180*	*0.010022*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.1%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
35 °C	SVP@Ts	0.008209	0.006157	0.004105	0.003079	0.002463	0.001642	0.001231	0.000821	Infinity	Type B
	F@Ts,Ps	0.007623	0.007783	0.007951	0.008042	0.008103	0.008214	0.008376	0.008575	Infinity	Type B
	SVP@Tc	0.008210	0.006163	0.004116	0.003093	0.002479	0.001660	0.001251	0.000841	Infinity	Type B
	F@Tc,Pc	0.007458	0.005599	0.003739	0.002810	0.002252	0.001508	0.001136	0.000764	Infinity	Type B
	Combined	*0.015765*	*0.012955*	*0.010535*	*0.009572*	*0.009107*	*0.008672*	*0.008633*	*0.008688*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.2%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
70 °C	SVP@Ts	0.002050	0.001539	0.001027	0.000771	0.000617	0.000411	0.000308	0.000206	Infinity	Type B
	F@Ts,Ps	0.003652	0.004423	0.005200	0.005593	0.005833	0.006177	0.006448	0.006745	Infinity	Type B
	SVP@Tc	0.002050	0.001539	0.001028	0.000772	0.000619	0.000414	0.000312	0.000209	Infinity	Type B
	F@Tc,Pc	0.003518	0.002642	0.001765	0.001326	0.001062	0.000711	0.000535	0.000359	Infinity	Type B
	Combined	*0.005841*	*0.005593*	*0.005680*	*0.005851*	*0.005993*	*0.006245*	*0.006485*	*0.006760*	*Infinity*	

Table 8

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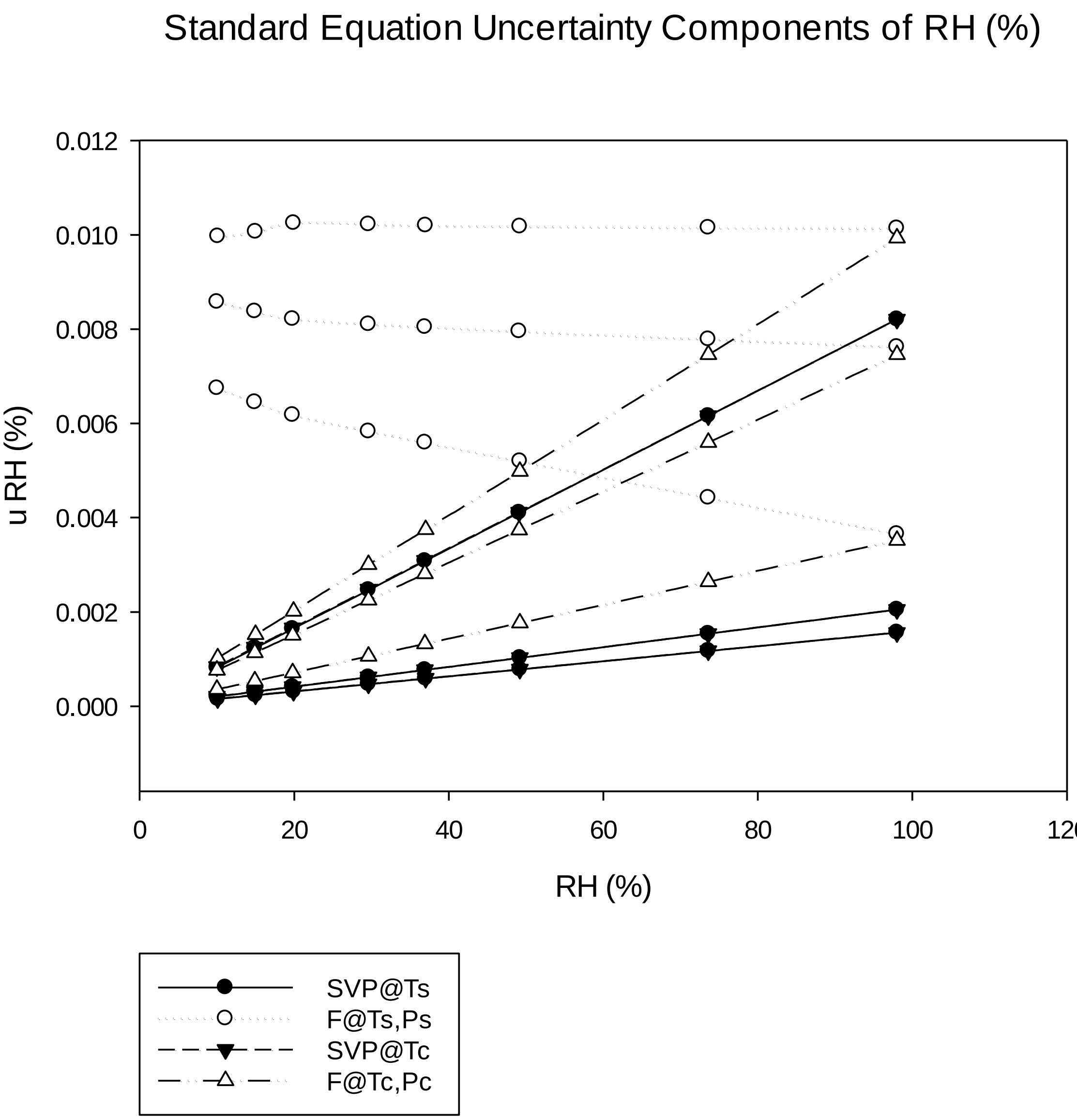

Figure 7

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The standard uncertainties, uT_D, components calculated using the associated equation uncertainty tables mentioned above are summarized in Table 9 and Figure 8.

Standard Equation Uncertainty Components of Dew Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.3 °C	**-4.1 °C**	**-9.4 °C**	**-13.0 °C**	**-15.7 °C**	**-20.4 °C**	**-23.6 °C**	**-27.9 °C**		
0 °C	SVP@Ts	0.000219	0.000212	0.000202	0.000196	0.000191	0.000182	0.000176	0.000167	Infinity	Type B
	F@Ts,Ps	0.001420	0.001834	0.002633	0.003405	0.004159	0.005967	0.007545	0.010657	Infinity	Type B
	SVP@Td	0.000219	0.000212	0.000203	0.000197	0.000192	0.000184	0.000179	0.000172	Infinity	Type B
	F@Td,Pc	0.001411	0.001630	0.001902	0.002068	0.002184	0.002349	0.002285	0.002200	Infinity	Type B
	Combined	*0.002026*	*0.002472*	*0.003260*	*0.003993*	*0.004705*	*0.006417*	*0.007887*	*0.010885*	*Infinity*	
		34.6 °C	**29.6 °C**	**22.7 °C**	**18.1 °C**	**14.6 °C**	**8.6 °C**	**4.5 °C**	**-1.1 °C**		
35 °C	SVP@Ts	0.001509	0.001452	0.001377	0.001326	0.001288	0.001223	0.001178	0.001115	Infinity	Type B
	F@Ts,Ps	0.001401	0.001836	0.002667	0.003464	0.004239	0.006116	0.008009	0.011650	Infinity	Type B
	SVP@Td	0.001541	0.001905	0.001997	0.001586	0.001295	0.000820	0.000522	0.000218	Infinity	Type B
	F@Td,Pc	0.001338	0.000923	0.001439	0.001607	0.001565	0.001493	0.001446	0.001457	Infinity	Type B
	Combined	*0.002899*	*0.003156*	*0.003881*	*0.004342*	*0.004874*	*0.006466*	*0.008240*	*0.011796*	*Infinity*	
		69.5 °C	**63.1 °C**	**54.4 °C**	**48.6 °C**	**44.3 °C**	**36.7 °C**	**31.6 °C**	**24.8 °C**		
70 °C	SVP@Ts	0.000482	0.000461	0.000434	0.000416	0.000403	0.000380	0.000365	0.000344	Infinity	Type B
	F@Ts,Ps	0.000859	0.001325	0.002197	0.003019	0.003810	0.005708	0.007623	0.011280	Infinity	Type B
	SVP@Td	0.000485	0.000507	0.000739	0.000936	0.001033	0.001382	0.001761	0.002186	Infinity	Type B
	F@Td,Pc	0.000844	0.001025	0.001063	0.001158	0.001542	0.001550	0.001042	0.001291	Infinity	Type B
	Combined	*0.001385*	*0.001810*	*0.002587*	*0.003392*	*0.004257*	*0.006086*	*0.007901*	*0.011568*	*Infinity*	

Table 9

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Standard Equation Uncertainty Components of Dew Point Temperature (°C)

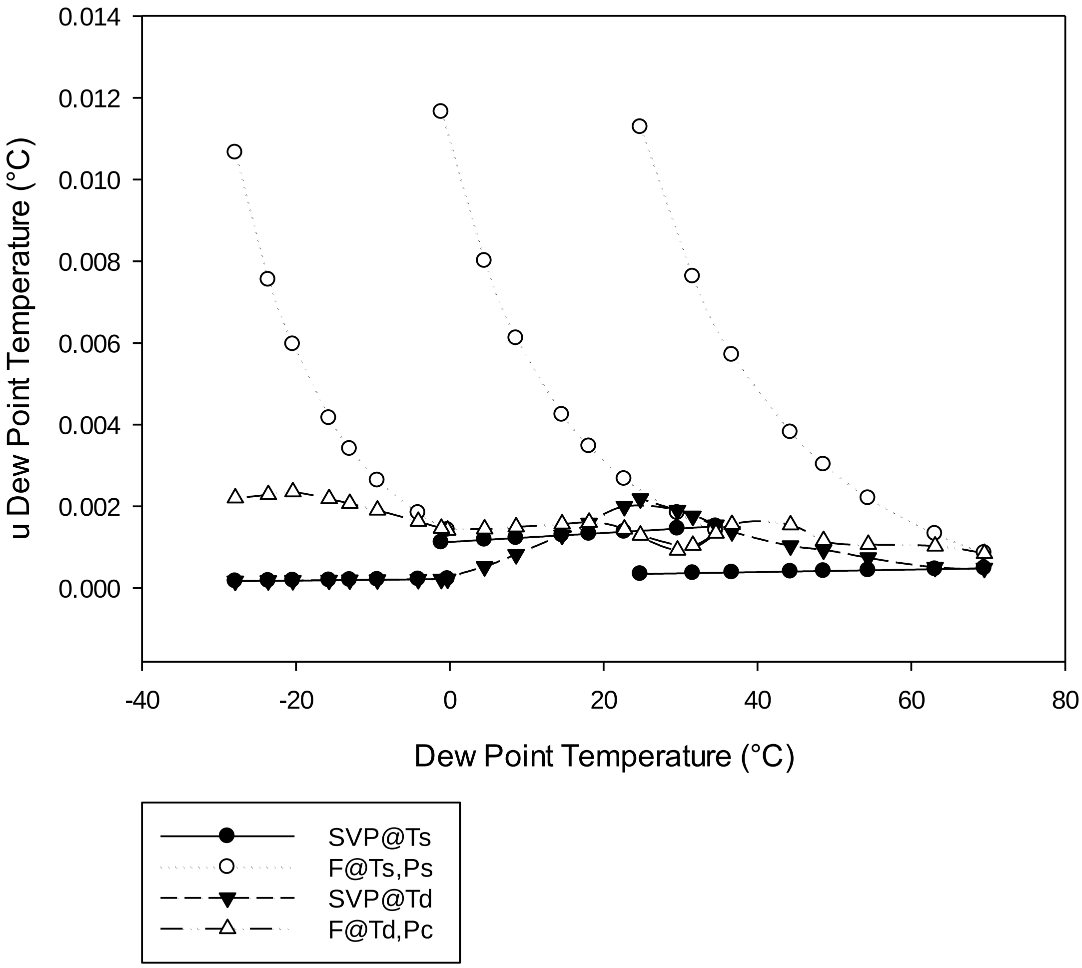

Figure 8

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The standard uncertainties, uT_F, components calculated using the associated equation uncertainty tables mentioned above are summarized in Table 10 and Figure 9.

Note: Any frost point value that is not possible is grayed out of the following table.

Standard Equation Uncertainty Components of Frost Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.2 °C	**-3.7 °C**	**-8.4 °C**	**-11.6 °C**	**-14.0 °C**	**-18.3 °C**	**-21.2 °C**	**-25.2 °C**		
0 °C	SVP@Ts	0.000193	0.000188	0.000181	0.000176	0.000173	0.000166	0.000161	0.000154	Infinity	Type B
	F@Ts,Ps	0.001254	0.001629	0.002359	0.003070	0.003766	0.005444	0.006920	0.009841	Infinity	Type B
	SVP@Tf	0.000353	0.002536	0.005335	0.006990	0.008070	0.009865	0.010939	0.012163	Infinity	Type B
	F@Tf,Pc	0.001244	0.001420	0.001644	0.001785	0.001886	0.002052	0.002094	0.002030	Infinity	Type B
	Combined	*0.001811*	*0.003337*	*0.006063*	*0.007843*	*0.009104*	*0.011454*	*0.013114*	*0.015777*	*Infinity*	
									-0.9 °C		
35 °C	SVP@Ts								0.000986	Infinity	Type B
	F@Ts,Ps								0.010297	Infinity	Type B
	SVP@Tf								0.000804	Infinity	Type B
	F@Tf,Pc								0.001280	Infinity	Type B
	Combined								*0.010453*	*Infinity*	
70 °C	SVP@Ts									Infinity	Type B
	F@Ts,Ps									Infinity	Type B
	SVP@Tf									Infinity	Type B
	F@Tf,Pc									Infinity	Type B
	Combined									*Infinity*	

Table 10

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Document: Model_2900_Uncertainty_Analysis (Rev3)
Author: Michael Hamilton
Date: February 2019

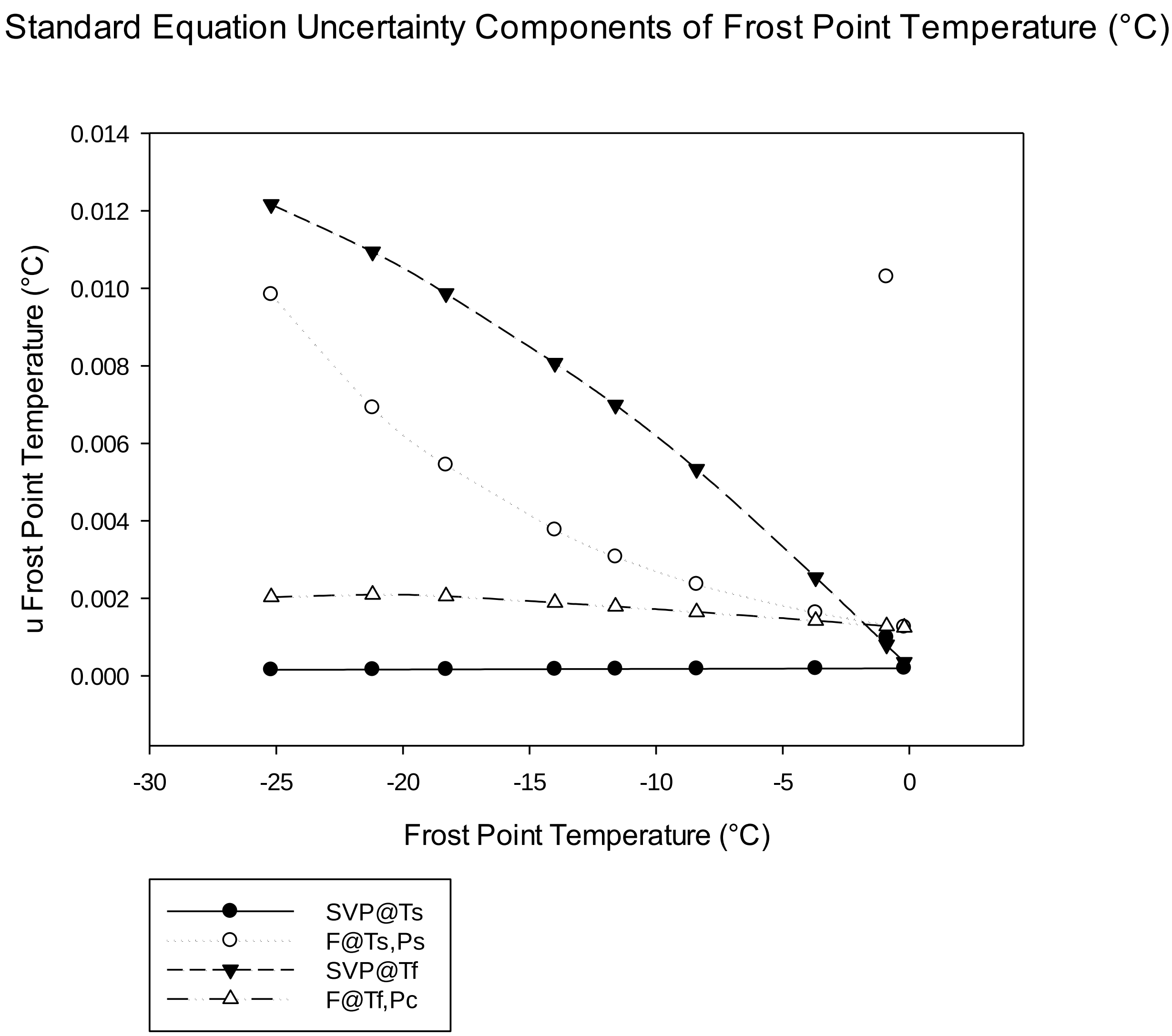

Figure 9

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3.4 Saturator Efficiency Uncertainty Contribution

All two-pressure humidity generators rely on the ability of the saturator to fully saturate the gas with water vapor as it passes from inlet to outlet. The Model 2900 humidity generator incorporates a pre-saturator device along with the saturator to assure the full saturation of the gas with water vapor. Why this design helps assure 100% saturation of the gas, there may still be small amounts of uncertainty.

Based on engineering research and development work on the Model 2900, the uncertainty component of % efficiency of saturation is determined to be

$$\eta_S = 99.93\%$$

The standard uncertainties, uRH, components calculated using the above associated % efficiency component are summarized in Table 11 and Figure 10.

Standard Saturator Efficiency Uncertainty Components of RH (%)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		98.0%RH	**73.6%RH**	**49.2%RH**	**37.0%RH**	**29.6%RH**	**19.9%RH**	**15.0%RH**	**10.1%RH**		
0 °C	η_S	0.068605	0.051515	0.034426	0.025881	0.020755	0.013920	0.010502	0.007086	Infinity	Type B
	Combined	*0.068605*	*0.051515*	*0.034426*	*0.025881*	*0.020755*	*0.013920*	*0.010502*	*0.007086*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.1%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
35 °C	η_S	0.068604	0.051501	0.034396	0.025844	0.020713	0.013871	0.010450	0.007029	Infinity	Type B
	Combined	*0.068604*	*0.051501*	*0.034396*	*0.025844*	*0.020713*	*0.013871*	*0.010450*	*0.007029*	*Infinity*	
		98.0%RH	**73.6%RH**	**49.2%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**		
70 °C	η_S	0.068606	0.051516	0.034410	0.025850	0.020713	0.013860	0.010433	0.007005	Infinity	Type B
	Combined	*0.068606*	*0.051516*	*0.034410*	*0.025850*	*0.020713*	*0.013860*	*0.010433*	*0.007005*	*Infinity*	

Table 11

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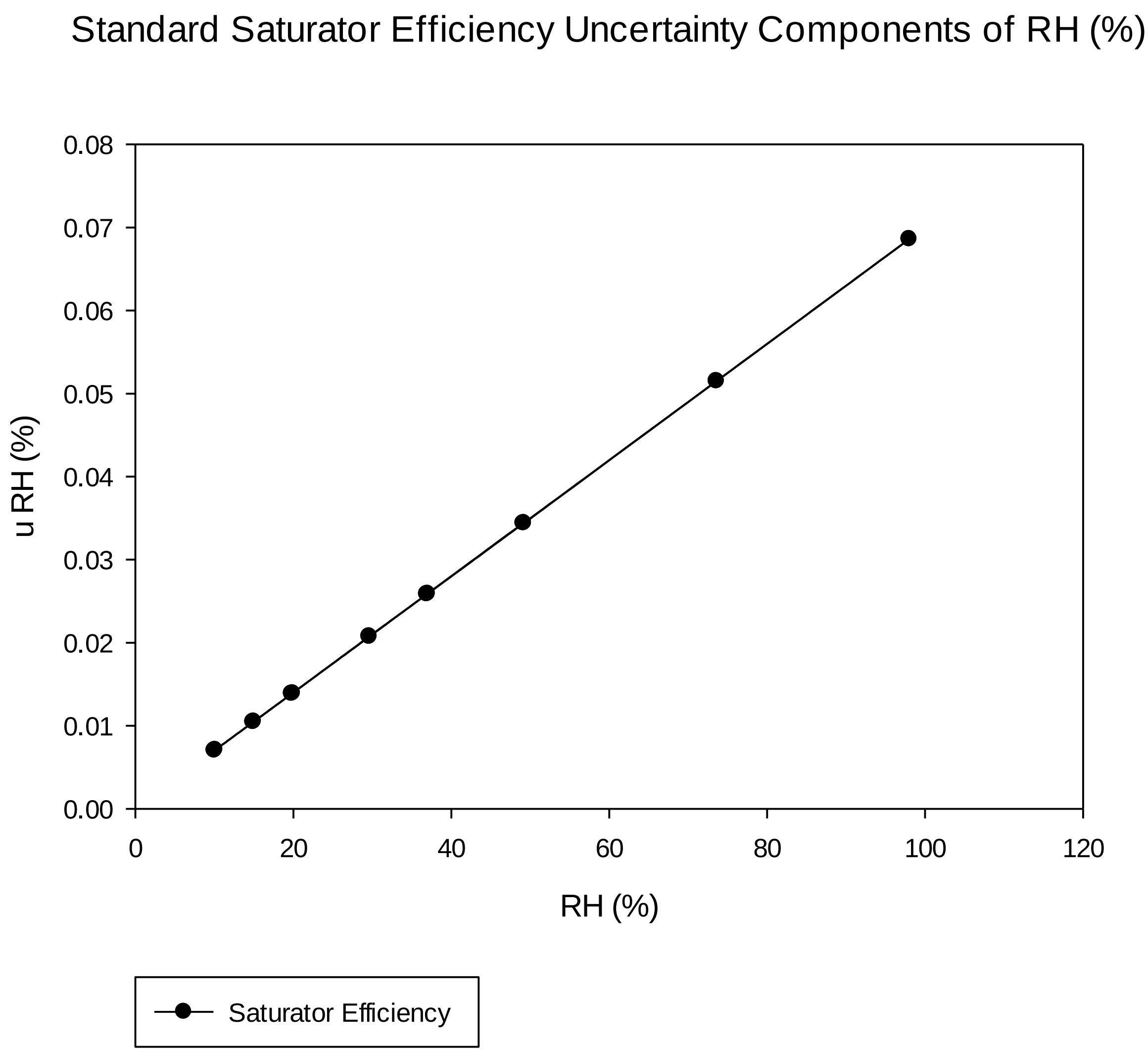

Figure 10

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The standard uncertainties, uT_D, components calculated using the above associated % efficiency component are summarized in Table 12 and Figure 11.

Standard Saturator Efficiency Uncertainty Components of Dew Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.3 °C	**-4.1 °C**	**-9.4 °C**	**-13.0 °C**	**-15.7 °C**	**-20.4 °C**	**-23.6 °C**	**-27.9 °C**		
0 °C	η_S	0.009613	0.009310	0.008909	0.008642	0.008444	0.008104	0.007877	0.007576	Infinity	Type B
	Combined	*0.009613*	*0.009310*	*0.008909*	*0.008642*	*0.008444*	*0.008104*	*0.007877*	*0.007576*	*Infinity*	
		34.6 °C	**29.6 °C**	**22.7 °C**	**18.1 °C**	**14.6 °C**	**8.6 °C**	**4.5 °C**	**-1.1 °C**		
35 °C	η_S	0.012613	0.012147	0.011536	0.011132	0.010835	0.010328	0.009992	0.009551	Infinity	Type B
	Combined	*0.012613*	*0.012147*	*0.011536*	*0.011132*	*0.010835*	*0.010328*	*0.009992*	*0.009551*	*Infinity*	
		69.5 °C	**63.1 °C**	**54.4 °C**	**48.6 °C**	**44.3 °C**	**36.7 °C**	**31.6 °C**	**24.8 °C**		
70 °C	η_S	0.016132	0.015435	0.014538	0.013954	0.013528	0.012807	0.012334	0.011716	Infinity	Type B
	Combined	*0.016132*	*0.015435*	*0.014538*	*0.013954*	*0.013528*	*0.012807*	*0.012334*	*0.011716*	*Infinity*	

Table 12

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Standard Saturator Efficiency Uncertainty Components of Dew Point Temperature (°C)

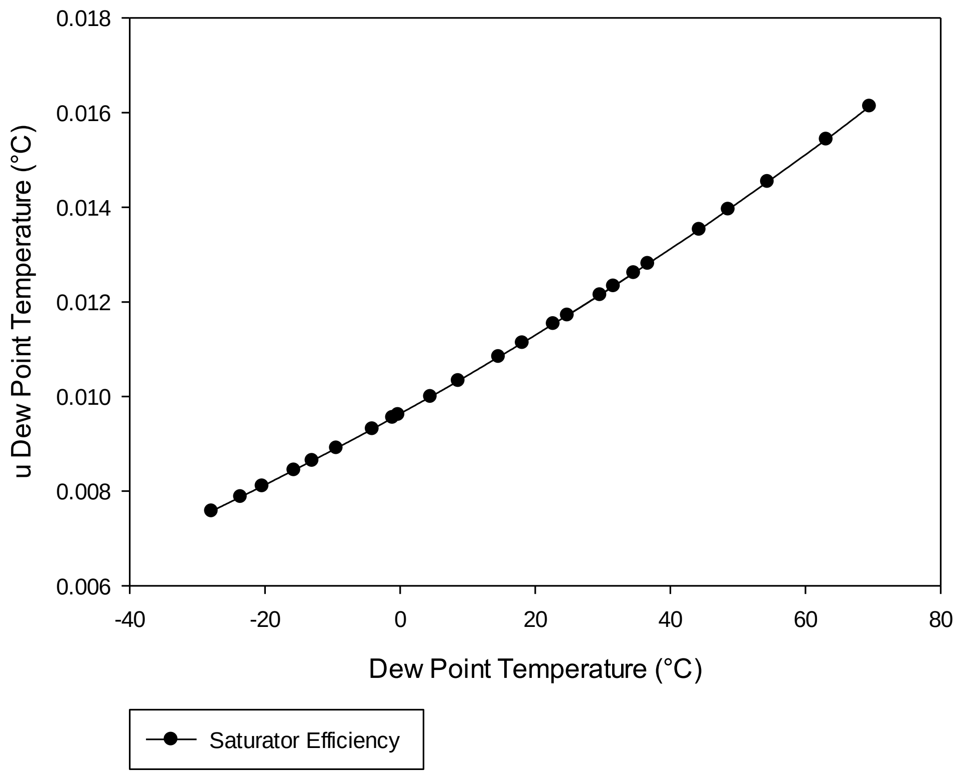

Figure 11

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The standard uncertainties, uT_F, components calculated using the above associated % efficiency component are summarized in Table 13 and Figure 12.

Note: Any frost point value that is not possible is grayed out of the following table.

Standard Saturator Efficiency Uncertainty Components of Frost Point Temperature (°C)											
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**	**Evaluation**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**		
		-0.2 °C	**-3.7 °C**	**-8.4 °C**	**-11.6 °C**	**-14.0 °C**	**-18.3 °C**	**-21.2 °C**	**-25.2 °C**		
0 °C	η_S	0.008484	0.008271	0.007985	0.007791	0.007646	0.007394	0.007224	0.006996	Infinity	Type B
	Combined	*0.008484*	*0.008271*	*0.007985*	*0.007791*	*0.007646*	*0.007394*	*0.007224*	*0.006996*	*Infinity*	
									-0.9 °C		
35 °C	η_S								0.008441	Infinity	Type B
	Combined								*0.008441*	*Infinity*	
70 °C	η_S									Infinity	Type B
	Combined									*Infinity*	

Table 13

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Standard Saturator Efficiency Uncertainty Components of Frost Point Temperature (°C)

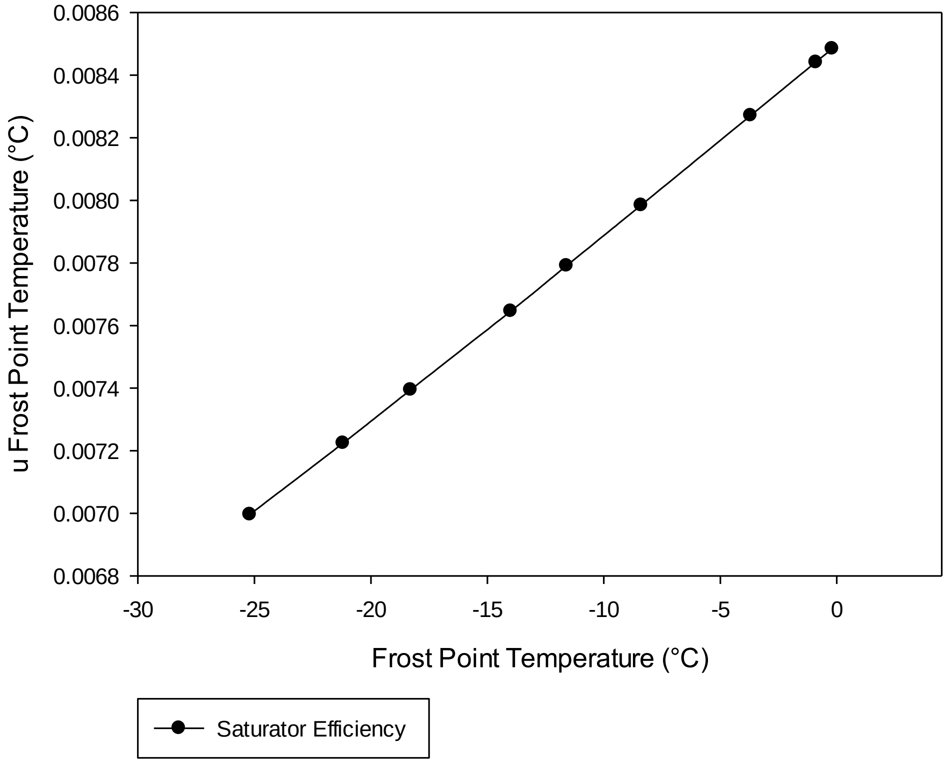

Figure 12

4.0 Combined Standard and Expanded Uncertainty

The combined standard uncertainty is obtained by the statistical combination of the individual standard uncertainty components of pressure, temperature, and equation in terms of relative humidity, dew point or frost point. Utilizing a confidence level of 95.45% and a coverage factor k=2, the expanded uncertainty, U, is expressed by multiplying the combined standard uncertainty by the coverage factor as show in the following formula

$$U = k * u_c \qquad (19)$$

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Using equations 6 and 19, the combined individual standard uncertainty components for pressure, temperature, equation and saturator efficiency, the total combined standard uncertainty (u) and the total combined expanded uncertainty (U) in terms of relative humidity RH (%) are summarized in Table 14 and Figure 13.

Uncertainty Components of RH (%)										
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**	
		98.0%RH	**73.6%RH**	**49.2%RH**	**37.0%RH**	**29.6%RH**	**19.9%RH**	**15.0%RH**	**10.1%RH**	
0 °C	Pressure	0.120822	0.068193	0.030601	0.017443	0.011351	0.005327	0.003210	0.001674	Infinity
	Temperature	0.199627	0.149893	0.100159	0.075292	0.060373	0.040481	0.030536	0.020593	Infinity
	Equation	0.014363	0.012703	0.011382	0.010895	0.010675	0.010454	0.010180	0.010022	Infinity
	Saturator Efficiency	0.068605	0.051515	0.034426	0.025881	0.020755	0.013920	0.010502	0.007086	Infinity
	Combined	*0.243643*	*0.173013*	*0.110828*	*0.082230*	*0.065715*	*0.044386*	*0.034010*	*0.024032*	*Infinity*
	Expanded (k=2)	*0.487285*	*0.346025*	*0.221657*	*0.164460*	*0.131429*	*0.088772*	*0.068020*	*0.048064*	
		98.0%RH	**73.6%RH**	**49.1%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**	
35 °C	Pressure	0.120915	0.068249	0.030627	0.017457	0.011359	0.005329	0.003208	0.001669	Infinity
	Temperature	0.156585	0.117545	0.078502	0.058979	0.047265	0.031647	0.023838	0.016030	Infinity
	Equation	0.015765	0.012955	0.010535	0.009572	0.009107	0.008672	0.008633	0.008688	Infinity
	Saturator Efficiency	0.068604	0.051501	0.034396	0.025844	0.020713	0.013871	0.010450	0.007029	Infinity
	Combined	*0.209987*	*0.145928*	*0.091622*	*0.067401*	*0.053619*	*0.036021*	*0.027609*	*0.019612*	*Infinity*
	Expanded (k=2)	*0.419973*	*0.291856*	*0.183245*	*0.134801*	*0.107238*	*0.072042*	*0.055218*	*0.039224*	
		98.0%RH	**73.6%RH**	**49.2%RH**	**36.9%RH**	**29.6%RH**	**19.8%RH**	**14.9%RH**	**10.0%RH**	
70 °C	Pressure	0.120759	0.068225	0.030644	0.017474	0.011372	0.005335	0.003210	0.001667	Infinity
	Temperature	0.132359	0.099407	0.066409	0.049892	0.039977	0.026749	0.020133	0.013516	Infinity
	Equation	0.005841	0.005593	0.005680	0.005851	0.005993	0.006245	0.006485	0.006760	Infinity
	Saturator Efficiency	0.068606	0.051516	0.034410	0.025850	0.020713	0.013860	0.010433	0.007005	Infinity
	Combined	*0.191944*	*0.131231*	*0.081028*	*0.059136*	*0.046823*	*0.031227*	*0.023802*	*0.016740*	*Infinity*
	Expanded (k=2)	*0.383888*	*0.262463*	*0.162055*	*0.118271*	*0.093646*	*0.062453*	*0.047605*	*0.033480*	

Table 14

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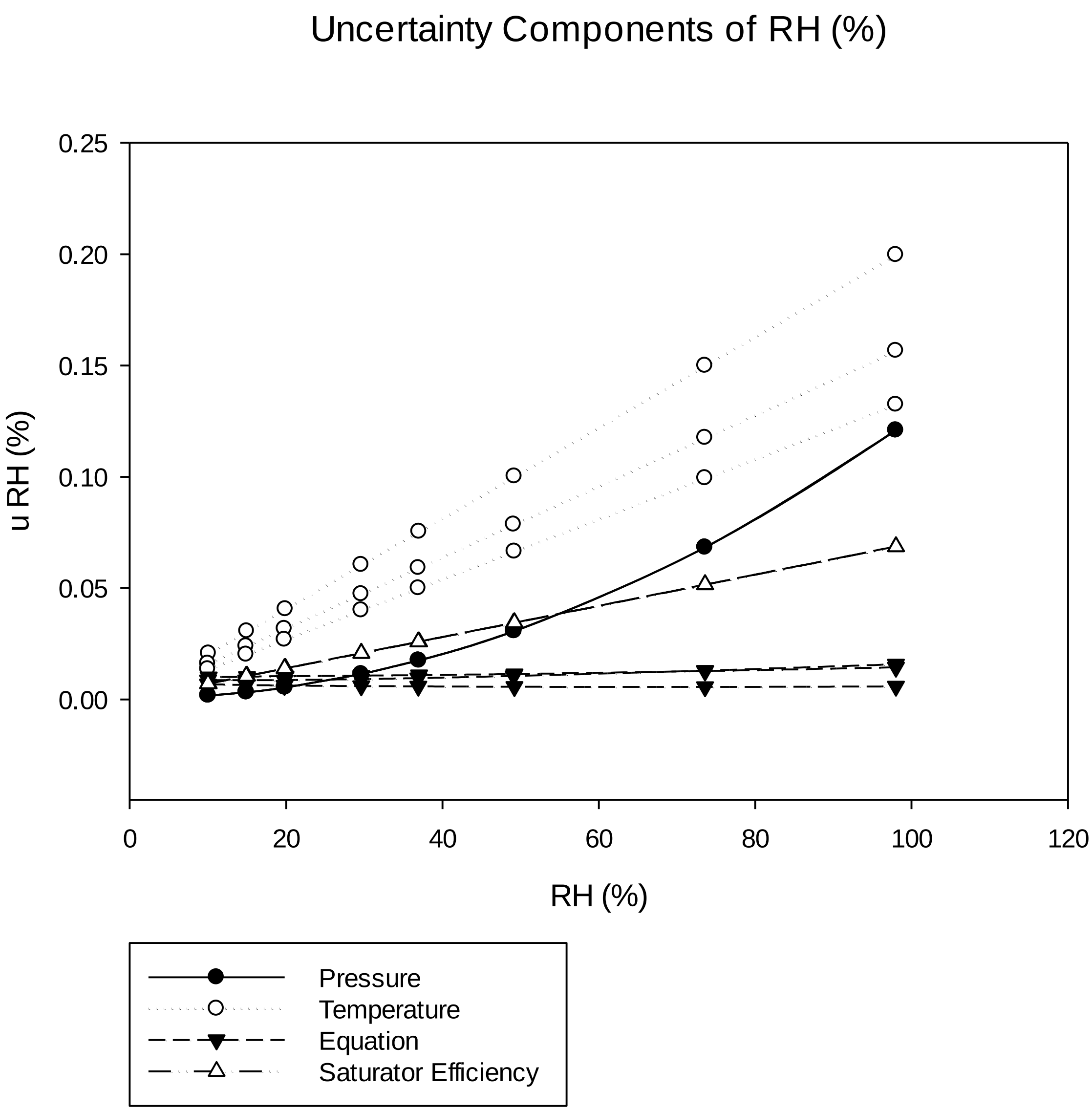

Figure 13

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Using equations 7 and 19, the combined individual standard uncertainty components for pressure, temperature, equation and saturator efficiency, the total combined standard uncertainty (u) and the total combined expanded uncertainty (U) in terms of dew point temperature Td (°C) are summarized in Table 15 and Figure 14.

Uncertainty Components of Dew Point Temperature (°C)										
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**	
		-0.3 °C	**-4.1 °C**	**-9.4 °C**	**-13.0 °C**	**-15.7 °C**	**-20.4 °C**	**-23.6 °C**	**-27.9 °C**	
0 °C	Pressure	0.016930	0.012324	0.007919	0.005824	0.004618	0.003101	0.002407	0.001789	Infinity
	Temperature	0.012673	0.012272	0.011740	0.011384	0.011120	0.010664	0.010358	0.009948	Infinity
	Equation	0.002026	0.002472	0.003260	0.003993	0.004705	0.006417	0.007887	0.010885	Infinity
	Saturator Efficiency	0.009613	0.009310	0.008909	0.008642	0.008444	0.008104	0.007877	0.007576	Infinity
	Combined	*0.023318*	*0.019881*	*0.017045*	*0.015942*	*0.015441*	*0.015172*	*0.015406*	*0.016675*	*Infinity*
	Expanded (k=2)	*0.046636*	*0.039763*	*0.034091*	*0.031885*	*0.030882*	*0.030344*	*0.030811*	*0.033349*	
		34.6 °C	**29.6 °C**	**22.7 °C**	**18.1 °C**	**14.6 °C**	**8.6 °C**	**4.5 °C**	**-1.1 °C**	
35 °C	Pressure	0.022230	0.016097	0.010272	0.007519	0.005942	0.003968	0.003067	0.002267	Infinity
	Temperature	0.012668	0.012199	0.011583	0.011175	0.010874	0.010359	0.010017	0.009563	Infinity
	Equation	0.002899	0.003156	0.003881	0.004342	0.004874	0.006466	0.008240	0.011796	Infinity
	Saturator Efficiency	0.012613	0.012147	0.011536	0.011132	0.010835	0.010328	0.009992	0.009551	Infinity
	Combined	*0.028673*	*0.023779*	*0.019693*	*0.018006*	*0.017167*	*0.016478*	*0.016658*	*0.018082*	*Infinity*
	Expanded (k=2)	*0.057346*	*0.047557*	*0.039386*	*0.036012*	*0.034334*	*0.032957*	*0.033316*	*0.036164*	
		69.5 °C	**63.1 °C**	**54.4 °C**	**48.6 °C**	**44.3 °C**	**36.7 °C**	**31.6 °C**	**24.8 °C**	
70 °C	Pressure	0.028396	0.020441	0.012947	0.009433	0.007428	0.004931	0.003796	0.002790	Infinity
	Temperature	0.012662	0.012124	0.011426	0.010970	0.010636	0.010067	0.009692	0.009199	Infinity
	Equation	0.001385	0.001810	0.002587	0.003392	0.004257	0.006086	0.007901	0.011568	Infinity
	Saturator Efficiency	0.016132	0.015435	0.014538	0.013954	0.013528	0.012807	0.012334	0.011716	Infinity
	Combined	*0.035055*	*0.028396*	*0.022721*	*0.020385*	*0.019221*	*0.018076*	*0.017970*	*0.019065*	*Infinity*
	Expanded (k=2)	*0.070110*	*0.056792*	*0.045442*	*0.040770*	*0.038441*	*0.036151*	*0.035940*	*0.038130*	

Table 15

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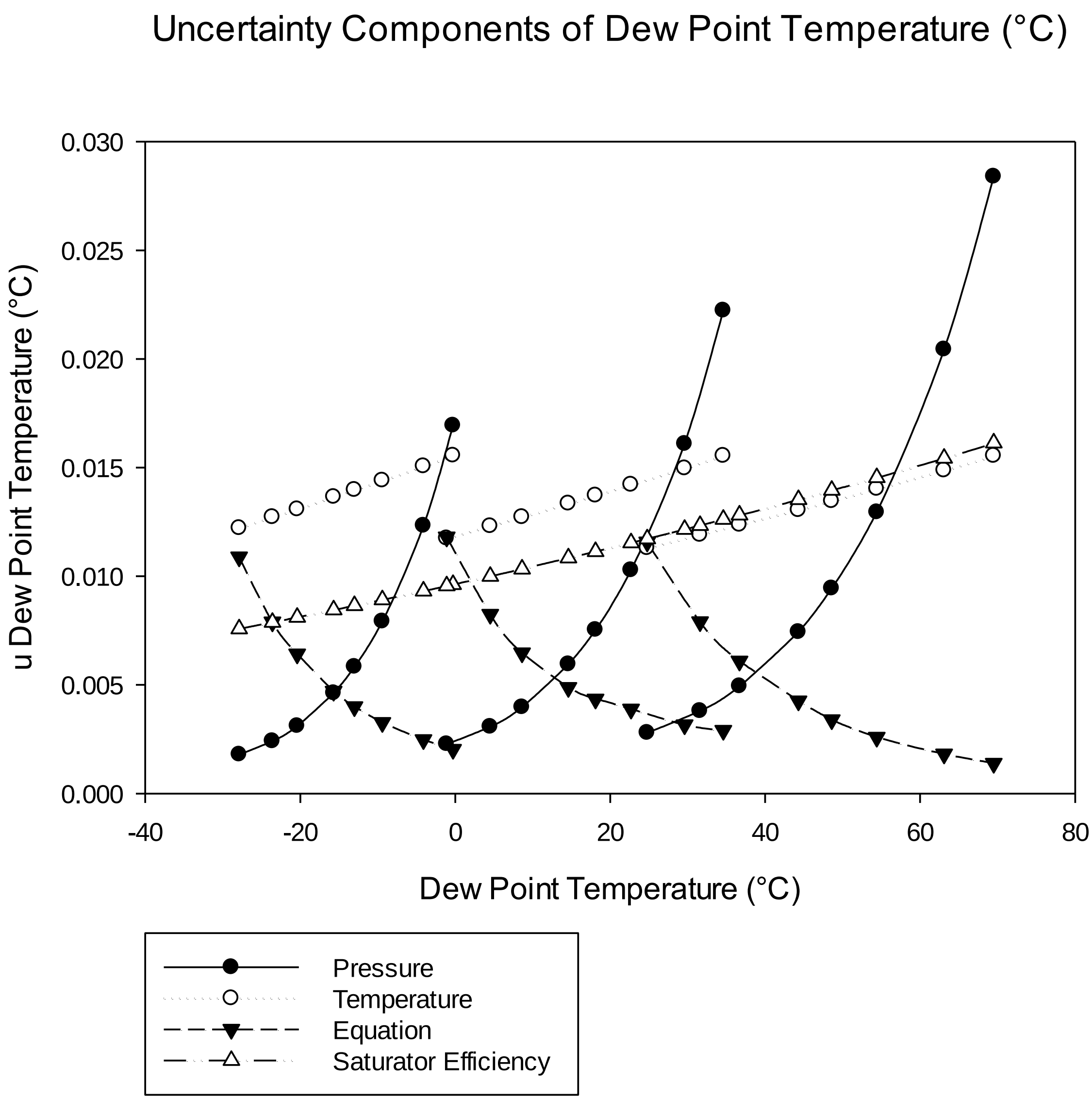

Figure 14

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Using equations 8 and 19, the combined individual standard uncertainty components for pressure, temperature, equation and saturator efficiency, the total combined standard uncertainty (u) and the total combined expanded uncertainty (U) in terms of frost point temperature Tf (°C) are summarized in Table 16 and Figure 15.

Note: Any frost point value that is not possible is grayed out of the following tables.

Uncertainty Components of Frost Point Temperature (°C)										
Saturation Temperature	**Description**	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								**Degrees of Freedom**
		15	**20**	**30**	**40**	**50**	**75**	**100**	**150**	
		-0.2 °C	**-3.7 °C**	**-8.4 °C**	**-11.6 °C**	**-14.0 °C**	**-18.3 °C**	**-21.2 °C**	**-25.2 °C**	
0 °C	Pressure	0.014942	0.010949	0.007097	0.005251	0.004181	0.002830	0.002207	0.001652	Infinity
	Temperature	0.013730	0.013380	0.012913	0.012596	0.012358	0.011942	0.011659	0.011274	Infinity
	Equation	0.001811	0.003337	0.006063	0.007843	0.009104	0.011454	0.013114	0.015777	Infinity
	Saturator Efficiency	0.008484	0.008271	0.007985	0.007791	0.007646	0.007394	0.007224	0.006996	Infinity
	Combined	*0.022069*	*0.019454*	*0.017822*	*0.017562*	*0.017651*	*0.018344*	*0.019104*	*0.020681*	*Infinity*
	Expanded (k=2)	*0.044139*	*0.038908*	*0.035645*	*0.035125*	*0.035302*	*0.036687*	*0.038207*	*0.041362*	
									-0.9 °C	
35 °C	Pressure								0.002003	Infinity
	Temperature								0.010372	Infinity
	Equation								0.010453	Infinity
	Saturator Efficiency								0.008441	Infinity
	Combined								*0.017092*	*Infinity*
	Expanded (k=2)								*0.034183*	
70 °C	Pressure									Infinity
	Temperature									Infinity
	Equation									Infinity
	Saturator Efficiency									Infinity
	Combined									*Infinity*
	Expanded (k=2)									

Table 16

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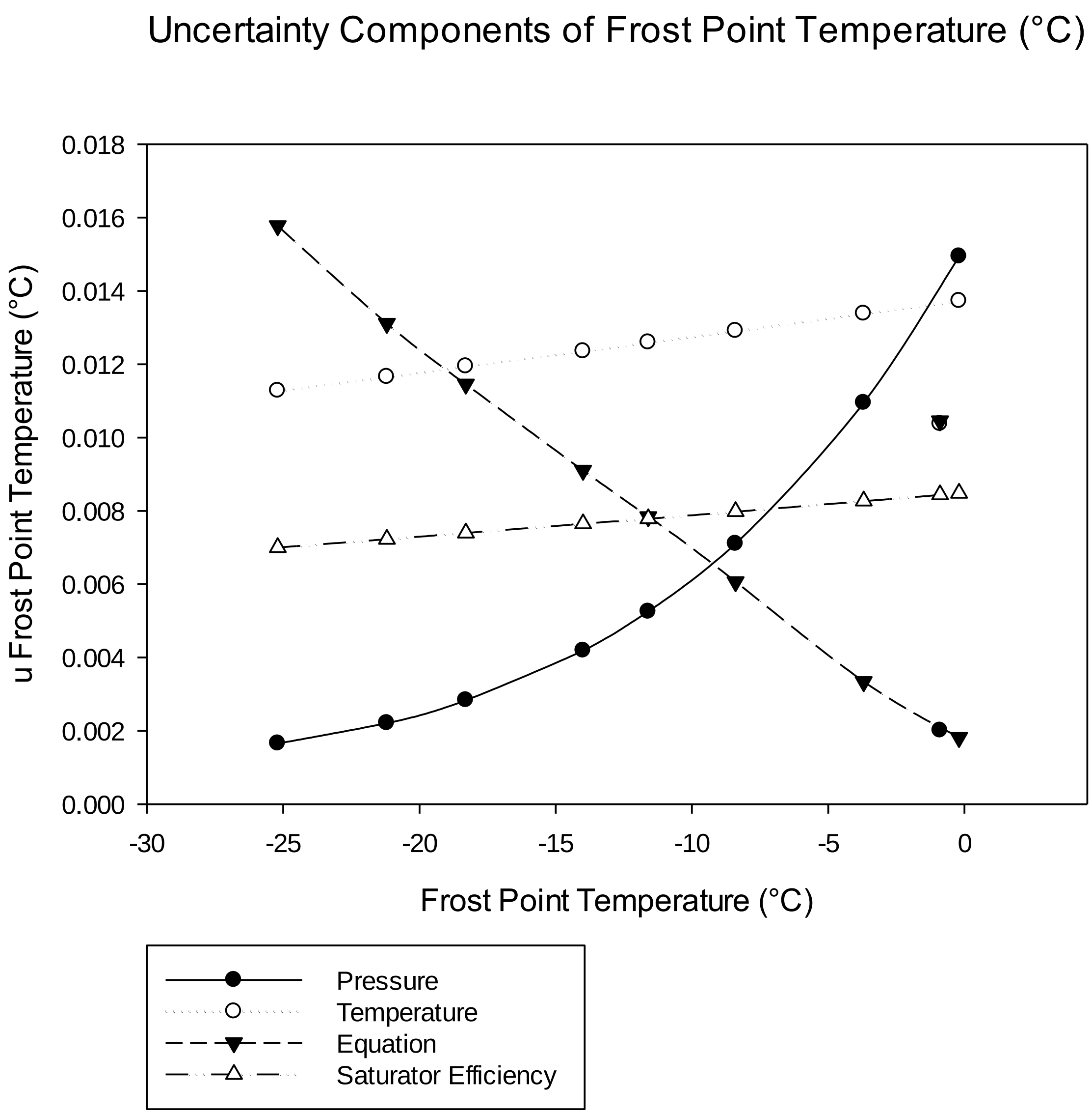

Figure 15

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5.0 Summary

To simplify the %RH uncertainty results, the following uncertainty specification statement is used to describe the RH (%) uncertainty for the Model 2900:

0.5% of reading RH

(using a coverage factor, k=2, at an approximate level of confidence of 95%)

A summary of the combined expanded uncertainty (U_{RH}) and uncertainty specification for RH (%) are shown in Table 17 and Figure 16.

Expanded Uncertainty of RH (%)								
Saturation Temperature	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**							
	15	**20**	**30**	**40**	**50**	**75**	**100**	**150**
	98.0 %RH	**73.6 %RH**	**49.2 %RH**	**37.0 %RH**	**29.6 %RH**	**19.9 %RH**	**15.0 %RH**	**10.0 %RH**
0 °C	0.487	0.346	0.222	0.164	0.131	0.089	0.068	0.048
35 °C	0.420	0.292	0.183	0.135	0.107	0.072	0.055	0.039
70 °C	0.384	0.262	0.162	0.118	0.094	0.062	0.048	0.033
0.5% Specification	0.490	0.368	0.246	0.185	0.148	0.100	0.075	0.050

Table 17

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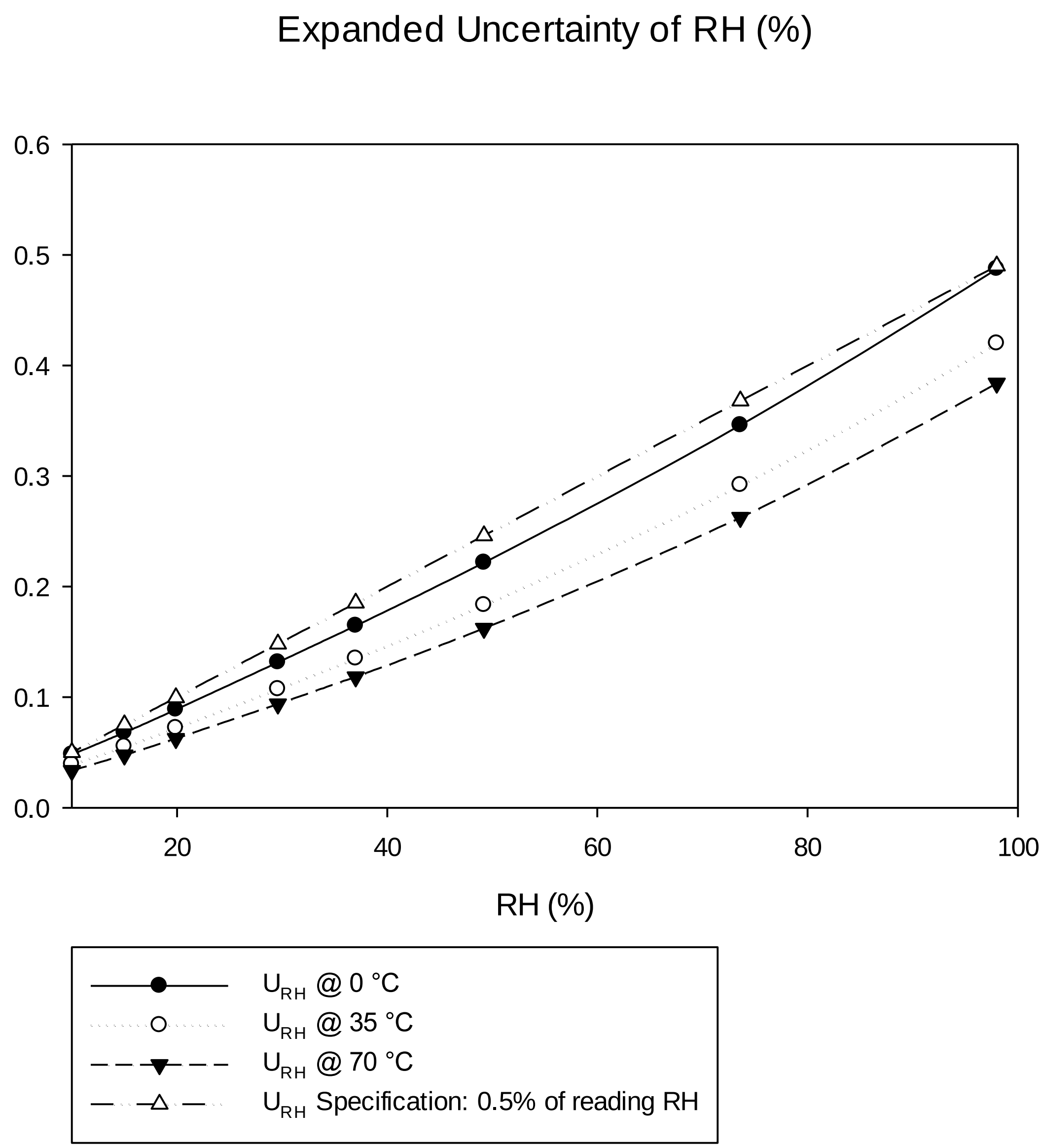

Figure 16

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To simplify the Dew Point Temperature uncertainty results, the following uncertainty specification statement is used to describe the Dew Point (°C) uncertainty (U_{Td}) for the Model 2900 over the range of 0 °C to 70 °C:

0.08 °C
(using a coverage factor, k=2, at an approximate level of confidence of 95%)

A summary of the combined expanded uncertainty and uncertainty specification for Dew Point Temperature (°C) over the range of 0 to 70 °C are shown in Table 18 and Figure 17.

Expanded Uncertainty of Dew Point Temperature (°C)								
Saturation Temperature	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**							
	15	**20**	**30**	**40**	**50**	**75**	**100**	**150**
	34.6 °C	**29.6 °C**	**22.7 °C**	**18.1 °C**	**14.6 °C**	**8.6 °C**	**4.5 °C**	
35 °C	0.057	0.048	0.039	0.036	0.034	0.033	0.033	
	69.5 °C	**63.1 °C**	**54.4 °C**	**48.6 °C**	**44.3 °C**	**36.7 °C**	**31.6 °C**	**24.8 °C**
70 °C	0.070	0.057	0.045	0.041	0.038	0.036	0.036	0.038
0.08 Specification	0.080	0.080	0.080	0.080	0.080	0.080	0.080	0.080

Table 18

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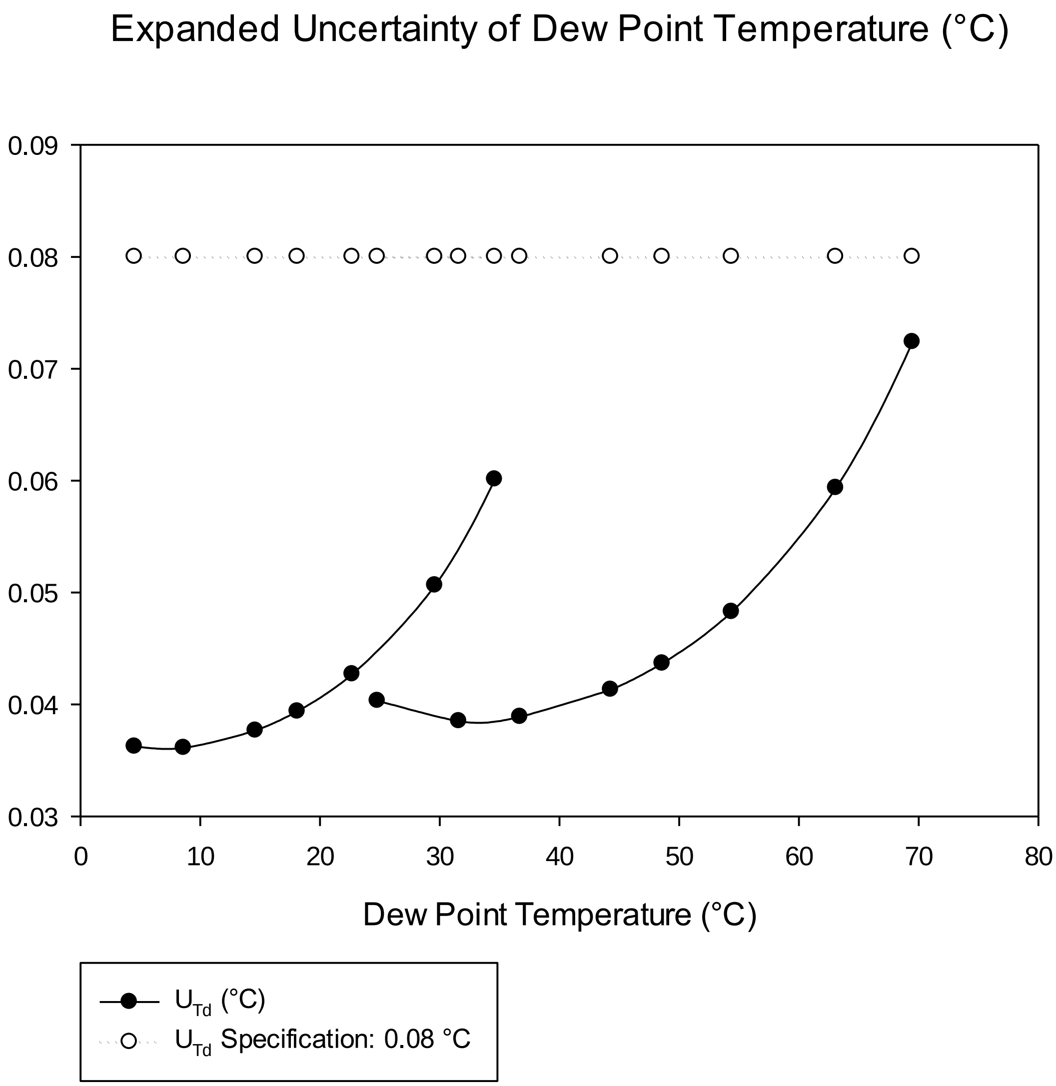

Figure 17

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To simplify the Dew/Frost Point Temperature uncertainty results, the following uncertainty specification statement is used to describe the Dew/Frost Point (°C) uncertainty (U_{Tf}) for the Model 2900 for Dew/Frost Point Temperatures below 0 °C:

0.05 °C

(using a coverage factor, k=2, at an approximate level of confidence of 95%)

A summary of the combined expanded uncertainty and uncertainty specification for Dew/Frost Point Temperature (°C) below 0 °C are shown in Table 18 and Figure 17.

Expanded Uncertainty of Dew/Frost Point Temperature (°C)									
Saturation Temperature	**Saturation Pressure Range (psia), Chamber pressure = 14.7 psia**								
	15	**20**	**30**	**40**	**50**	**75**	**100**	**150**	
	-0.3 °C	**-4.1 °C**	**-9.4 °C**	**-13.0 °C**	**-15.7 °C**	**-20.4 °C**	**-23.6 °C**	**-27.9 °C**	**Td**
0 °C	0.047	0.040	0.034	0.032	0.031	0.030	0.031	0.033	
	-0.2 °C	**-3.7 °C**	**-8.4 °C**	**-11.6 °C**	**-14.0 °C**	**-18.3 °C**	**-21.2 °C**	**-25.2 °C**	**Tf**
0 °C	0.041	0.036	0.032	0.032	0.032	0.034	0.036	0.039	
								-1.1 °C	**Td**
35 °C								0.036	
								-0.9 °C	**Tf**
35 °C								0.032	
0.05 Specification	0.050	0.050	0.050	0.050	0.050	0.050	0.050	0.050	

Table 19

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Document: Model_2900_Uncertainty_Analysis (Rev3)
Author: Michael Hamilton
Date: February 2019

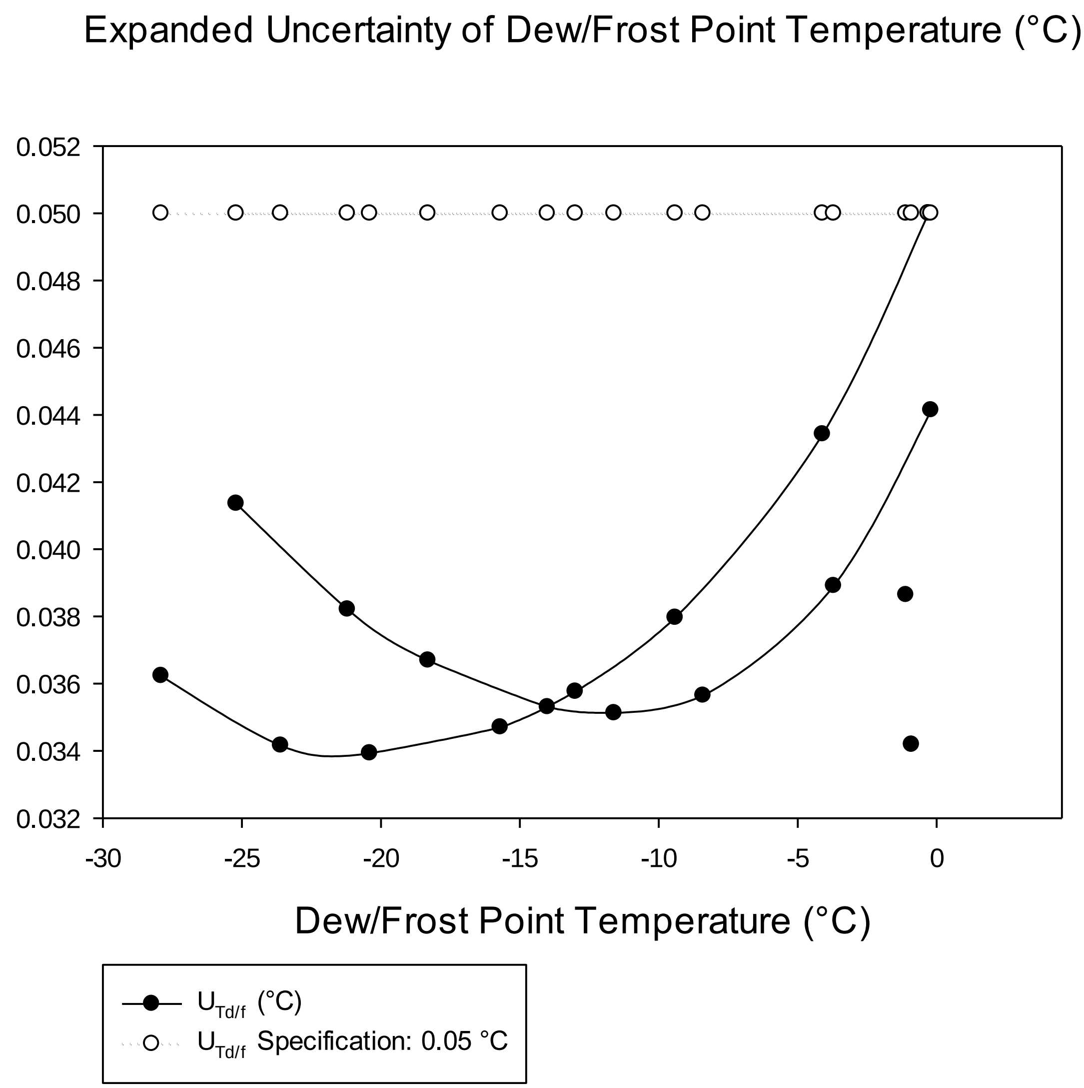

Figure 18

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