

Dew/Frost Point Uncertainty Analysis of the Model 3900 Two-Temperature, Two-Pressure Low Humidity Generator

Revision E
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1 Introduction

Described here is the Dew Point and Frost Point Uncertainty Analysis, following NIST Guideline 1297¹, for a Model 3900 Humidity Generator, manufactured by Thunder Scientific Corporation, that combines the NIST developed and proven two-temperature and two-pressure humidity generation principles.^{2,3} Generating gas of a known dew point or frost point temperature in a system of this type does not require direct measurements of the water vapor content of the gas. Rather, the generated dew point and/or frost point temperature is derived from the measurements of saturation temperature, saturation pressure, and the pressure at the point of use, commonly referred to as either test pressure or chamber pressure. For the purposes of this analysis, the terms ‘test pressure’ and ‘chamber pressure’ are synonymous with each other.

2 Defining Equations

2.1 Common Equations

The following equations of Wexler/Hardy⁷ for saturation vapor pressure, enhancement factor, and temperature (from saturation vapor pressure) are common and fundamental to most humidity calculations presented here.

2.1.1 Saturation Vapor Pressure over Water, e

Saturation vapor pressure over *water* at a given ITS-90 temperature is defined by the formula ^[7]

$$e = \exp\left(\sum_{i=0}^6 g_i T^{i-2} + g_7 \ln T\right) \quad (1)$$

where e is the saturation vapor pressure, in Pascals, over liquid water in the pure phase,
 T is the temperature in Kelvin,

and

$$\begin{aligned} g_0 &= -2.8365744 \cdot 10^3 \\ g_1 &= -6.028076559 \cdot 10^3 \\ g_2 &= 1.954263612 \cdot 10^1 \\ g_3 &= -2.737830188 \cdot 10^{-2} \\ g_4 &= 1.6261698 \cdot 10^{-5} \\ g_5 &= 7.0229056 \cdot 10^{-10} \\ g_6 &= -1.8680009 \cdot 10^{-13} \\ g_7 &= 2.7150305 \end{aligned}$$

2.1.2 Saturation Vapor Pressure over Ice, e

Saturation vapor pressure over *ice* at a given temperature is defined by the formula ^[7]

$$e = \exp\left(\sum_{i=0}^4 k_i T^{i-1} + k_5 \ln T\right) \quad (2)$$

where e is the saturation vapor pressure, in Pascals, over ice in the pure phase
 T is the temperature in Kelvin

and $k_0 = -5.8666426 \cdot 10^3$
 $k_1 = 2.232870244 \cdot 10^1$
 $k_2 = 1.39387003 \cdot 10^{-2}$
 $k_3 = -3.4262402 \cdot 10^{-5}$
 $k_4 = 2.7040955 \cdot 10^{-8}$
 $k_5 = 6.7063522 \cdot 10^{-1}$

2.1.3 Enhancement Factors

The ‘effective’ saturation vapor pressure over water or ice in the presence of other gases differs from the ideal saturation vapor pressures given in equations 1 and 2. The effective saturation vapor pressure is related to the ideal by

$$\acute{e} = e \cdot f \quad (3)$$

where \acute{e} is the ‘effective’ saturation vapor pressure
 e is the ideal saturation vapor pressure (as given in equation 1 or 2)
and f is the enhancement factor.

The enhancement factor, for an air-water vapor mixture, is determined at a given temperature and pressure from the formula^[7]

$$f = \exp\left[\alpha\left(1 - \frac{e}{P}\right) + \beta\left(\frac{P}{e} - 1\right)\right] \quad (4)$$

with $\alpha = \sum_{i=0}^3 a_i T^i$ (5)

and $\beta = \exp\left(\sum_{i=0}^3 b_i T^i\right)$ (6)

where f is the enhancement factor
 e is the ideal saturation vapor pressure (as given in equation 1 or 2)
 P is pressure in the same units as e
 T is temperature in Kelvin
and a_i, b_i depend on temperature range and are given as

for water

223.15 to 273.15 K (-50 to 0 °C)

$a_0 = -5.5898101 \cdot 10^{-2}$
 $a_1 = 6.7140389 \cdot 10^{-4}$
 $a_2 = -2.7492721 \cdot 10^{-6}$
 $a_3 = 3.8268958 \cdot 10^{-9}$
 $b_0 = -8.1985393 \cdot 10^1$
 $b_1 = 5.8230823 \cdot 10^{-1}$

273.15 to 373.15 K (0 to 100 °C)

$a_0 = -1.6302041 \cdot 10^{-1}$
 $a_1 = 1.8071570 \cdot 10^{-3}$
 $a_2 = -6.7703064 \cdot 10^{-6}$
 $a_3 = 8.5813609 \cdot 10^{-9}$
 $b_0 = -5.9890467 \cdot 10^1$
 $b_1 = 3.4378043 \cdot 10^{-1}$

$$b_2 = -1.6340527 \cdot 10^{-3}$$

$$b_3 = 1.6725084 \cdot 10^{-6}$$

$$b_2 = -7.7326396 \cdot 10^{-4}$$

$$b_3 = 6.3405286 \cdot 10^{-7}$$

for ice

173.15 to 223.15 K (-100 to -50 °C)

$$a_0 = -7.4712663 \cdot 10^{-2}$$

$$a_1 = 9.5972907 \cdot 10^{-4}$$

$$a_2 = -4.1935419 \cdot 10^{-6}$$

$$a_3 = 6.2038841 \cdot 10^{-9}$$

$$b_0 = -1.0385289 \cdot 10^2$$

$$b_1 = 8.5753626 \cdot 10^{-1}$$

$$b_2 = -2.8578612 \cdot 10^{-3}$$

$$b_3 = 3.5499292 \cdot 10^{-6}$$

223.15 to 273.15 K (-50 to 0 °C)

$$a_0 = -7.1044201 \cdot 10^{-2}$$

$$a_1 = 8.6786223 \cdot 10^{-4}$$

$$a_2 = -3.5912529 \cdot 10^{-6}$$

$$a_3 = 5.0194210 \cdot 10^{-9}$$

$$b_0 = -8.2308868 \cdot 10^1$$

$$b_1 = 5.6519110 \cdot 10^{-1}$$

$$b_2 = -1.5304505 \cdot 10^{-3}$$

$$b_3 = 1.5395086 \cdot 10^{-6}$$

2.1.4 Temperature from Saturation Vapor Pressure

Equations 1 and 2 are easily solved for saturation vapor pressure over water or ice for a given saturation temperature. However, if vapor pressure is known while temperature is the unknown desired quantity, the solution immediately becomes complicated and must be solved by iteration. For ease of computation, the following inverse equation is provided. This equation is generally used to find the dew point or frost point temperature when the vapor pressure of a gas has been determined. When vapor pressure is known, use the water coefficients to obtain dew point, and use the ice coefficients to obtain frost point.

$$T = \frac{\sum_{i=0}^3 c_i (\ln e)^i}{\sum_{i=0}^3 d_i (\ln e)^i} \quad (7)$$

where T is the temperature in Kelvin
and e is the saturation vapor pressure in Pascals

with coefficients

for water

$$c_0 = 2.0798233 \cdot 10^2$$

$$c_1 = -2.0156028 \cdot 10^1$$

$$c_2 = 4.6778925 \cdot 10^{-1}$$

$$c_3 = -9.2288067 \cdot 10^{-6}$$

$$d_0 = 1$$

$$d_1 = -1.3319669 \cdot 10^{-1}$$

$$d_2 = 5.6577518 \cdot 10^{-3}$$

$$d_3 = -7.5172865 \cdot 10^{-5}$$

for ice

$$c_0 = 2.1257969 \cdot 10^2$$

$$c_1 = -1.0264612 \cdot 10^1$$

$$c_2 = 1.4354796 \cdot 10^{-1}$$

$$c_3 = 0$$

$$d_0 = 1$$

$$d_1 = -8.2871619 \cdot 10^{-2}$$

$$d_2 = 2.3540411 \cdot 10^{-3}$$

$$d_3 = -2.4363951 \cdot 10^{-5}$$

2.2 Dew Point and Frost Point Determination

2.2.1 Definitions of Terms

T_s *Saturation Temperature.* The temperature at which the gas is fully saturated with water vapor, and is most often made by a direct measurement of the temperature of the saturator itself. T_s is in Kelvin, t_s is in °C.

T_c *Chamber Temperature.* The temperature of the gas in the test chamber, or in the device under test, at the location of the humidity sensor. T_c is in Kelvin, t_c is in °C.

- T_d*** *Dew Point Temperature.* The temperature to which a gas must be cooled in order to just begin condensing in the form of liquid dew. While contrary to common sense, liquid dew can form in a meta-stable state at temperatures below freezing (called super-cooled dew). Thus, dew point temperatures below 0 °C are quite common and reproducible. While dew point and frost point exhibit identical vapor pressures, dew point and frost point temperatures are not the same, except at 0.01 °C which is the triple point of water. T_d is in Kelvin; t_d is in °C.
- T_f*** *Frost Point Temperature.* The temperature to which a gas must be cooled in order to just begin condensing in the form of frost or ice. Frost point only exists at temperatures below freezing ($t_f \leq 0.01$ °C). While frost point and dew point exhibit identical vapor pressures, frost point and dew point temperatures are not the same, except at 0.01 °C which is the triple point of water. T_f is in Kelvin, t_f is in °C.
- P_s*** *Saturator Pressure.* (As it applies to the humidity generators described in this document, *Saturation Pressure* is synonymous with *Saturator Pressure*.) The total pressure in the saturator, measured at the *final point of saturation* (generally the saturator outlet). This is an absolute (not gauge) measurement. P_s is in Pascals.
- P_c*** *Chamber Pressure.* The total pressure as measured in the test chamber, or at the device under test, at the location of the humidity sensor. This is also referred to throughout this document as Test Pressure. This is an absolute (not gauge) measurement. P_c is in Pascals.
- e_s*** *Saturation Vapor Pressure at the Saturation Temperature.* The partial pressure of the water vapor in the saturator, as determined by measurement of the saturation temperature. Regardless of the total pressure of the saturator, e_s is dependent on saturation temperature only and further assumes that full saturation is actually being achieved. For temperatures above freezing, e_s is computed as *Saturation Vapor pressure over Water*. For temperatures below freezing, e_s is generally computed as *Saturation Vapor Pressure over Ice*. e_s is expressed in Pascals.
- e_c*** *Saturation Vapor Pressure at the Chamber Temperature.* The maximum possible partial pressure of water vapor that could exist in the test chamber, if the gas were fully saturated with water vapor at the chamber temperature. Regardless of the total pressure of the chamber, e_c is dependent on chamber temperature only. For temperatures above freezing, e_c is computed as *Saturation Vapor pressure over Water*. For temperatures below freezing, e_c is generally computed as *Saturation Vapor Pressure over Ice*. However, when using e_c in the computation of %RH, but only when doing so in accordance with the World Meteorological Organization (WMO) adopted guidelines, e_c is to be computed with respect to water for all temperature conditions, even those below freezing. e_c is expressed in Pascals.
- e_d*** *Saturation Vapor Pressure at the Dew/Frost Point Temperature (also known as Dew Point Vapor Pressure, Frost Point Vapor Pressure, and partial water vapor pressure).* The partial pressure of the water vapor at the dew point temperature and computed with respect to liquid water, or at the frost point temperature and computed with respect to ice. Where frost point exists (at all temperatures below freezing), dew point vapor pressure calculated at the dew point temperature with respect to liquid water and frost point vapor pressure calculated at the frost point temperature with respect to ice are always equal and synonymous terms. e_d is expressed in Pascals.
- f_s*** *Enhancement Factor at Saturation Temperature and Saturation Pressure.* The enhancement factor corrects for the slight non-ideal behavior of water vapor when admixed with other gases. The *effective* saturation vapor pressure that results under saturation at any given temperature and pressure condition is determined by computing the product of the saturation vapor pressure and the enhancement factor.

f_c *Enhancement Factor at Chamber Temperature and Chamber Pressure.* The enhancement factor corrects for the slight non-ideal behavior of water vapor when admixed with other gases. The *effective* saturation vapor pressure that results under saturation at any given temperature and pressure condition is determined by computing the product of the saturation vapor pressure and the enhancement factor.

f_d *Enhancement Factor at Dew/Frost Point Temperature and Chamber Pressure.* The enhancement factor corrects for the slight non-ideal behavior of water vapor when admixed with other gases. The *effective* dew/frost point vapor pressure that results under saturation at any given temperature and pressure condition is determined by computing the product of the dew/frost point vapor pressure and the enhancement factor.

2.2.2 Dew Point Temperature, T_d

Dew point temperature is the temperature to which a gas must be cooled in order to just begin condensing water vapor in the form of dew. Dew point temperature in this system is obtained with the following iterative steps.

- A. The vapor pressure e_s at the saturation temperature is calculated with equation 1 or equation 2. To get e_s from this equation, the saturation temperature T_s is used for T . When $T_s > 0$, equation 1 is used. When $T_s < 0$, specific knowledge of the state of the water (whether liquid or ice) is needed. While it is possible for the water in a saturator to remain liquid for a short time when below 0 °C, liquid water in a saturator operating below 0 °C will eventually freeze into a state of ice. (A saturator that has been operating at or below -5 °C for more than an hour is most often expected to be operating in a state of ice.) Once frozen into ice, the water remains in that state as long as the saturation temperature remains below 0 °C. When the saturator is operating in a state of ice, equation 2 is required.
- B. The enhancement factor f_s at the saturation temperature and saturation pressure is calculated using equation 4. To get f_s from this equation, calculations are performed using $e = e_s$, $P = P_s$, and $T = T_s$. Equation 4 must be used with the correct coefficients (relative to the correct temperature range for water or ice) based upon saturation temperature T_s and specific knowledge of the state of the water in the saturator.
- C. An educated guess is made at the dew/frost point enhancement factor, f_d . Setting $f_d = 1$ is a suitable first guess.
- D. The dew/frost point vapor pressure e_d of the gas is computed with the two-pressure, two-temperature relationship

$$e_d = e_s \cdot \frac{f_s}{f_d} \cdot \frac{P_c}{P_s} \quad (8)$$

- E. Dew point temperature T_d is calculated from dew/frost point vapor pressure e_d using equation 7 and the coefficients for water. To get T_d from this equation, calculations are performed using $e = e_d$.
- F. The dew/frost point enhancement factor f_d is calculated using equation 4 and the coefficients for water of the appropriate range (based on the value of T_d). To get f_d from this equation, calculations are performed using $e = e_d$, $P = P_c$, and $T = T_d$.
- G. The dew point temperature, T_d , converges to the proper value by iterating steps D through F several times as necessary.

2.2.3 Frost Point Temperature, T_f

Frost point temperature is the temperature to which a gas must be cooled in order to just begin condensing water vapor in the form of frost or ice. Frost point only exists at temperatures below freezing.

Note that generating a frost point does not require ice in the saturator, nor does generating a dew point temperature require liquid water in the saturator. While the state of the saturator is required for proper application of the equations and their calculations, the generator is ultimately controlling at some specified vapor pressure. Dew point temperature and frost point temperature share the same vapor pressure and enhancement factor, but have different numeric values of temperature. All vapor pressures that correspond to a dew point temperature at or below 0.01 °C also have a corresponding frost point temperature. At a dew point temperature of 0.01 °C, frost point temperature is equal to dew point temperature. For all values below that down to -50 °C, the two diverge from each other with dew point temperature always lower in value than the corresponding frost point temperature. Dew point temperature and frost point temperature, while different from each other in numeric value, are equally valid methods of expressing the same vapor pressure. Dew point temperatures below -50 °C are considered unobtainable as no vapor pressure data for water is available for temperature below -50 °C, while frost point temperature remains valid down to -100 °C as vapor pressure data with respect to ice is available.

Frost point is obtained with the following iterative steps.

- A. The vapor pressure e_s at the saturation temperature is calculated with equation 1 or equation 2. To get e_s from this equation, the saturation temperature T_s is used for T . When $T_s > 0$, equation 1 is used. When $T_s < 0$, specific knowledge of the state of the water (whether liquid or ice) is needed. While it is possible for the water in a saturator to remain liquid for a short time when below 0 °C, liquid water in a saturator operating below 0 °C will eventually freeze into a state of ice. (A saturator that has been operating at or below -5 °C for more than an hour is most often expected to be operating in a state of ice.) Once frozen into ice, the water remains in that state as long as the saturation temperature remains below 0 °C. When the saturator is operating in a state of ice, equation 2 is required.
- B. The enhancement factor f_s at the saturation temperature and saturation pressure is calculated using equation 4. To get f_s from this equation, calculations are performed using $e = e_s$, $P = P_s$, and $T = T_s$. Equation 4 must be used with the correct coefficients (relative to the correct temperature range for water or ice) based upon saturation temperature T_s and specific knowledge of the state of the water in the saturator.
- C. An educated guess is made at the dew/frost point enhancement factor, f_d . Setting $f_d = 1$ is a suitable first guess.
- D. The dew/frost point vapor pressure e_d of the gas is using equation 8.
- E. Frost point temperature T_f is calculated from dew/frost point vapor pressure e_d using equation 7 and the coefficients for ice. To get T_f from this equation, calculations are performed using $e = e_d$.
- F. The dew/frost point enhancement factor f_d is calculated using equation 4 and the coefficients for ice of the appropriate range (based on the value of T_f). To get f_d from this equation, calculations are performed using $e = e_d$, $P = P_c$, and $T = T_f$.
- G. The frost point temperature, T_f , converges to the proper value by iterating steps D through F several times as necessary.

3 Uncertainty

To analyze the overall expanded uncertainty in generated dew point and/or frost point temperature, the uncertainties associated with temperatures and pressures must be determined, along with other possible sources of uncertainty. These individual components of uncertainty must then be statistically combined to form the Combined Uncertainty. The Expanded Uncertainty is then determined by multiplying the Combined Uncertainty by a suitable coverage factor, k , based on the desired confidence level.

Due to the complexity associated with the computations, and the iterative requirement to reach a final solution, algebraic methods involving partial derivatives of the underlying equations prove difficult. Rather, a more straightforward approach will be taken that utilizes a table of sensitivity coefficients at various temperature and pressure combinations. The table will identify the sensitivity of generated dew and frost point temperature to uncertainty in the saturation temperature, saturation pressure, and test pressure. Construction of the sensitivity tables will then allow straight forward determination of the uncertainty in dew and frost point temperature due to:

- uncertainty in saturation pressure which includes
 - measurement uncertainty
 - measurement hysteresis
 - measurement resolution
- uncertainty in test pressure which includes
 - measurement uncertainty
 - measurement hysteresis
 - measurement resolution
- uncertainty in saturation temperature which includes
 - measurement uncertainty
 - measurement hysteresis
 - measurement resolution
- uncertainty contribution from saturator efficiency
- uncertainty in vapor pressure
- uncertainty in enhancement factors
- uncertainty contribution due to adsorption, desorption, and permeation

3.1 Creation of Sensitivity Coefficients at Various Saturation Temperatures and Pressures

Due to the complexity in developing algebraic solutions, a computer science approach is taken to determine these values. The calculation steps of sections 2.2.2 and 2.2.3 along with equations 1 through 8 are programmed into a computer that determines dew point and frost point temperatures using variable inputs of saturation temperature T_s , saturation pressure P_s , and test pressure P_c . A sensitivity coefficient is then determined by calculating the first derivative of the dew/frost point equation for each of the given inputs of saturation temperature, saturation pressure, and chamber pressure at various combinations.

Table 1. Dew Point & Frost Point Sensitivity Coefficients

Humidity Generator Conditions			Frost Point Sensitivity Coefficients			Dew Point Sensitivity Coefficients		
Nominal FP [°C]	T_s [°C]	P_s [kPa]	$\Delta FP/\Delta T_s$ [°C/°C]	$\Delta FP/\Delta P_s$ [°C/kPa]	$\Delta FP/\Delta P_c$ [°C/kPa]	$\Delta DP/\Delta T_s$ [°C/°C]	$\Delta DP/\Delta P_s$ [°C/kPa]	$\Delta DP/\Delta P_c$ [°C/kPa]
-95	-80	1668.93	0.843	0.003	0.051			

	-79.05	2000	0.834	0.002					
-90	-80	597.029	0.897	0.009	0.054				
	-75	1414.74	0.849	0.003					
	-73.04	2000	0.829	0.002					
	-80	101.325	1.000	0.060					
-80	-72.37	344.74	0.925	0.017	0.060				
	-70	498.099	0.903	0.012					
	-60.91	2000	0.821	0.003					
	-70	101.325	1.000	0.066					
-70	-61.54	344.74	0.921	0.019	0.066				
	-60	427.266	0.907	0.015					
	-50	1667.27	0.823	0.004					
	-48.66	2000	0.812	0.003					
-60	-60	101.325	1.000	0.072	0.072				
	-50.66	344.74	0.917	0.021					
	-50	374.63	0.912	0.019					
	-40	1277.76	0.832	0.005					
-50	-36.28	2000	0.803	0.003	0.079				
	-50	101.325	1.000	0.079					
	-40	334.27	0.915	0.024					
	-39.73	344.74	0.913	0.023					
-40	-30	1020.07	0.839	0.008	0.087				
	-23.76	2000	0.794	0.004					
	-40	101.325	1.000	0.087			1.057	0.092	0.092
	-30	302.6	0.919	0.029			0.971	0.030	
	-28.76	344.74	0.909	0.025			0.961	0.027	
-20	839.95	0.846	0.010		0.894	0.011			
-11.1	2000	0.785	0.004		0.830	0.004			
-30	-30	101.325	1.000	0.094	0.094			0.101	
	-20	277.21	0.922	0.034			1.074		0.101
	-17.73	344.74	0.905	0.027			0.990		0.037
	-10	708.82	0.851	0.013			0.973		0.030
	0	1723.92	0.738	0.005			0.915		0.014
	1.96	2000	0.682	0.004			0.793		0.006
-20	-20	101.325	1.000	0.102	0.102			0.112	
	-10	256.5	0.925	0.040			1.093		0.112
	-6.66	344.74	0.901	0.030			1.010		0.044
	0	610.31	0.805	0.017			0.985		0.033
	10	1248.98	0.695	0.008			0.879		0.018
	16.91	2000	0.657	0.005			0.760		0.009
-10	-10	101.325	1.000	0.111	0.111			0.123	
	0	239.37	0.872	0.047			1.112		0.123
	5.08	344.74	0.784	0.032			0.970		0.052
	10	484.44	0.754	0.023			0.873		0.036
	17	770.59	0.713	0.014			0.838		0.025
0	0	101.325						0.135	
	10	204.24					1.000		0.135
	17	323.42					0.922		0.067
10	10	101.325						0.147	
	17	160.19					1.000		0.147
						0.946	0.093	0.147	

Notice that in the table of sensitivity coefficients, nominal frost and dew point values are shown with a variety of saturation temperature and pressure combinations. For each specific frost point or dew point temperature listed, an attempt was made to include the maximum possible saturation pressure, minimum saturation pressure, and a pressure equal to the switch-over point between the high and low range saturation pressure transducer. In addition, the lowest and highest possible saturation temperature was also listed. For all of the above calculations, a common test pressure (standard atmospheric pressure) is assumed.

3.2 Uncertainty Contribution from Pressure

Determining the uncertainty in generated output based on saturation pressure and test pressure requires knowledge of uncertainty in the pressure measurement, pressure hysteresis, and measurement resolution of the saturator and test pressure transducers.

3.2.1 Pressure Measurement

This system utilizes three absolute pressure transducers. One is used for the test pressure measurement. While it has a full scale of 0 to 345 kPa absolute, it is generally used only for the barometric pressure range. For the purposes of this analysis, standard barometric pressure of 101.325 kPa is assumed. The other two pressure transducers are of different ranges and are used for the measurement of saturation pressure. For low saturation pressures (those below 345 kPa), the low range transducer is used. For higher saturation pressure, where the uncertainty in saturation pressure is of a lesser concern, a higher range transducer is used.

Pressure measurement uncertainty was analyzed from the data collected during annual calibration of the pressure transducers. During calibration, each transducer was tested at no fewer than 3 points over its specific range using a total system calibration approach. With this approach, the transducers remain electrically connected to the system allowing the pressure transducer, the measuring electronics, and the displayed data to be calibrated as a complete system rather than as individual components. Data gathered during the calibration is system rather than component data. The combined data from several years of calibration history were used in the computation of statistical standard deviations. For each of the pressure transducers (which includes the measurement electronics and display), the standard deviation, σ_P , from the desired mean values were determined to be:

$$\begin{aligned}\sigma_P &= 0.069 \text{ kPa (0.01 psia) for } P < 345 \text{ kPa (<50 psia)} \\ \sigma_P &= 0.276 \text{ kPa (0.04 psia) for } P > 345 \text{ kPa (>50 psia)}\end{aligned}$$

Using normal distribution, the pressure uncertainties normalized to one sigma are equivalent to the standard deviation values given above for each of the transducers.

$$\begin{aligned}uP_{c[meas]} &= 0.069 \text{ kPa (0.01 psia)} \\ uP_{s[meas]} &= 0.069 \text{ kPa (0.01 psia) for } P < 345 \text{ kPa (<50 psia)} \\ &= 0.276 \text{ kPa (0.04 psia) for } P > 345 \text{ kPa (>50 psia)}\end{aligned}$$

The statistical standard deviations calculated from the calibration history also have an uncertainty component from the Mensor PCS400 pressure standard used during the calibration process. The uncertainty of the Mensor PCS400 pressure standard for the low and high-pressure ranges are as follows:

$$\begin{aligned}uP_{c[std]} &= 0.046 \text{ kPa} \\ uP_{s[std]} &= 0.046 \text{ kPa for } P < 345 \text{ kPa (<50 psia)} \\ &= 0.228 \text{ kPa for } P > 345 \text{ kPa (>50 psia)}\end{aligned}$$

3.2.2 Pressure Hysteresis

The system most often operates for hours, days, or weeks on end at one saturation pressure. When a change in saturation pressure is made, the new condition is again maintained for hours, days, or weeks. The test pressure transducer generally monitors changes only in the barometric pressure. Since both the saturator and test pressure measurements result in very slow moving, nearly steady state conditions, hysteresis in pressure measurement is nearly imperceptible. However, the affect of any hysteresis must still be considered in the overall analysis. Based on rectangular distribution, hysteresis in pressure measurement normalized to one sigma, is estimated to be

$$\begin{aligned}uP_{c[hyst]} &= 0.035 / \sqrt{3} \text{ kPa (0.005/}\sqrt{3} \text{ psia)} \\ &= 0.020 \text{ kPa (0.003 psia)}\end{aligned}$$

$$\begin{aligned}uP_{s[hyst]} &= 0.035 / \sqrt{3} \text{ kPa (0.005/}\sqrt{3} \text{ psia) for } P < 345 \text{ kPa (50 psia)} \\ &= 0.020 \text{ kPa (0.003 psia) for } P < 345 \text{ kPa (50 psia)} \\ &= 0.172 / \sqrt{3} \text{ kPa (0.025/}\sqrt{3} \text{ psia) for } P > 345 \text{ kPa (50 psia)} \\ &= 0.099 \text{ kPa (0.014 psia) for } P > 345 \text{ kPa (50 psia)}\end{aligned}$$

3.2.3 Pressure Resolution

The analog to digital conversion process resolves 1 part in 25000 over the range of each of the pressure transducers. The resolution is then computed as

$$resolution_p = (TransducerRange)/25000$$

The resolution for each of the transducers is then

$$\begin{aligned}resolution_{pc} &= 345 \text{ kPa} / 25000 \\ &= 0.014 \text{ kPa (0.002 psia)}\end{aligned}$$

$$\begin{aligned}resolution_{ps} &= 345 \text{ kPa} / 25000 \\ &= 0.014 \text{ kPa for } P < 345 \text{ kPa (0.002 psia for } P < 50 \text{ psia)} \\ &= 2068 \text{ kPa} / 25000 \\ &= 0.083 \text{ kPa for } P > 345 \text{ kPa (0.012 psia for } P > 50 \text{ psia)}\end{aligned}$$

Based on a rectangular distribution of the half-interval of resolution, the uncertainty of pressure due to resolution normalized to one sigma is computed by multiplying the resolution by $0.5/\sqrt{3}$. The uncertainty of pressure due to resolution is then

$$uP_{[res]} = resolution_p * 0.5/\sqrt{3}$$

$$uP_{c[res]} = 0.004 \text{ kPa (0.0006 psia)}$$

$$\begin{aligned}uP_{s[res]} &= 0.004 \text{ kPa for } P < 345 \text{ kPa (0.0006 psia for } P < 50 \text{ psia)} \\ &= 0.024 \text{ kPa for } P > 345 \text{ kPa (0.0035 psia for } P > 50 \text{ psia)}\end{aligned}$$

3.2.4 Summary of Pressure Uncertainties

Pressure uncertainties of various types may be combined statistically using the following

$$(uP)^2 = (uP_{[meas]})^2 + (uP_{[std]})^2 + (uP_{[hyst]})^2 + (uP_{[res]})^2 \dots$$

Uncertainty data in tables 2 through 4 are combined in that manner.

Table 2. Test Pressure Uncertainty, uP_c

Source	Type	Uncertainty [kPa]
$uP_{c[meas]}$	A	0.069
$uP_{c[std]}$	B	0.023
$uP_{c[hyst]}$	B	0.020
$uP_{c[res]}$	B	0.004
uP_c		0.076

Table 3. Saturation Pressure Uncertainty, uP_s (for $P_s < 345$ kPa)

Source	Type	Uncertainty [kPa]
$uP_{s[meas]}$	A	0.069
$uP_{s[std]}$	B	0.023
$uP_{s[hyst]}$	B	0.020
$uP_{s[res]}$	B	0.004
uP_s ($P_s < 345$ kPa)		0.076

Table 4. Saturation Pressure Uncertainty, uP_s (for $P_s > 345$ kPa)

Source	Type	Uncertainty [kPa]
$uP_{s[meas]}$	A	0.276
$uP_{s[std]}$	B	0.077
$uP_{s[hyst]}$	B	0.099
$uP_{s[res]}$	B	0.024
uP_s ($P_s > 345$ kPa)		0.304

3.3 Uncertainty Contribution from Temperature

Determining the uncertainty in generated output based on saturation temperature requires knowledge of uncertainty in the measurement, the resolution, and the self-heating of the saturation thermometer.

Saturation temperature is measured by a 1k ohm thermistor to measure the saturation temperature range of -80 to +17 °C. Temperature measurement uncertainty was analyzed from the data collected during annual calibration of the saturator temperature probe. During calibration, the saturation thermometer was tested at no fewer than 3 points over its measurement range using a total system calibration approach. With this approach, the thermometer remains electrically connected to the system allowing the temperature probe, the measuring electronics, and the displayed data to be calibrated as a complete system rather than as individual components. Data gathered during the calibration is system rather than component data. The combined data from several years of calibration history were used in the computation of statistical standard deviations. The standard deviation of the saturation thermometer system, σ_T , from the desired mean values were determined to be:

$$\sigma_{T [meas]} = 0.023 \text{ }^{\circ}\text{C}$$

Using normal distribution, the temperature measurement uncertainty normalized to one sigma is equivalent to the standard deviation value given above for the saturation thermometer.

$$uT_{s[meas]} = 0.023 \text{ }^{\circ}\text{C}$$

The statistical standard deviations calculated from the calibration history also have an uncertainty component from the Hart 1560 Black Stack & 5626 PRT temperature standard used during the calibration process. The uncertainty of the Hart 1560 Black Stack & 5626 PRT temperature standard is as follows:

$$uT_{s[std]} = 0.006 \text{ }^{\circ}\text{C}$$

3.3.1 Temperature Resolution

The analog to digital conversion process which transforms the saturation thermometer resistance into digital values resolves to 0.002 °C. Based on a rectangular distribution of the half-interval, the uncertainty component of saturation temperature resolution is then

$$\begin{aligned} uT_{s[res]} &= 0.01 * 0.5/\sqrt{3} \\ &= 0.003 \end{aligned}$$

3.3.3 Self Heating

The saturation temperature probe is generally calibrated and checked in a well-stirred fluid bath. In use, it is also immersed directly within a pumped fluid surrounding the saturator. Since the conditions of calibration and use are very similar, both immersed within moving fluid, the self heating is considered almost negligible. However, an estimate of uncertainty will be applied.

$$uT_{s[self\ heating]} = 0.005 \text{ }^{\circ}\text{C}$$

3.3.4 Thermal Lag

The saturator is of a stacked plate design, constructed completely of stainless steel, sealed and immersed in a pumped fluid medium. Direction of the fluid flow is counter to that of the saturator gas stream. The temperature of the pumped fluid medium is controlled to the desired saturation temperature and measured by the saturation temperature probe. Given adequate time, the saturator outlet is assumed to come into thermal equilibrium with the average temperature of the pumped fluid medium. However, during times of temperature transition, the saturator plates will lag the temperature of the fluid by up to several degrees. No attempt will be made here to predict the uncertainty associated with thermal lag. However it will be assumed that adequate time is allowed for the saturator to regain thermal equilibrium with the pumped fluid medium prior to relying on the data from the generator. Lag times of 30 minutes to 1 hour are not considered uncommon. When approaching the final value, the rate of change is very slow and becomes difficult to detect on the instrument under test. Therefore, an estimate of uncertainty will be applied.

$$uT_{s[\text{thermal lag}]} = 0.01 \text{ }^{\circ}\text{C}$$

3.3.5 Thermal Gradients

Design of the saturator is that of a counter-flow design where the fluid medium flows in a direction opposite that of the gas stream being saturated. Thermal gradients do exist within the saturator from inlet to outlet. Controlling the direction of this gradient is important to proper saturation. The temperature of the fluid is measured and controlled at the point it enters the saturator cavity, which is the same point that the saturated gas stream exits the saturator due to the counter flow design. The temperature will be slightly higher at the fluid exit point, which is also the gas entry point. Provided the saturator is of sufficient thermal capacity and effective path length, complete thermal transfer between the gas flowing in one direction and the fluid flowing in the opposite direction will ensure that the exiting gas has reached thermal equilibrium with the entering fluid and is therefore at fluid temperature. An estimate of uncertainty will be applied.

$$uT_{s[\text{gradient}]} = 0.005 \text{ }^{\circ}\text{C}$$

[It is believed that the design of the saturator reduces any negative affects that a temperature gradient might otherwise cause if uncontrolled or improperly directed. Furthermore, it is believed that this design actually improves the ability of the saturator to fully saturate the gas stream with water vapor, thereby improving saturator efficiency.]

3.3.6 Temperature Control Stability

Temperature stability relates to the ability of the temperature control system to maintain a constant temperature in the pumped fluid medium, and ultimately in the saturator itself. The saturator temperature control system maintains the saturation temperature fluid at the setpoint with a standard deviation of 0.02 °C. Using normal distribution, the uncertainty in temperature relating to control stability normalized to one sigma is equivalent to the standard deviation. It is given therefore as

$$uT_{s[\text{stability}]} = 0.02 \text{ }^{\circ}\text{C}$$

3.3.7 Summary of Temperature Uncertainties

Temperature uncertainties of various types may be combined statistically using the following

$$(uT)^2 = (uT_{[meas]})^2 + (uT_{[resolution]})^2 + (uT_{[std]})^2 \dots$$

Uncertainty data in table 5 is combined in that manner.

Table 5. Saturation Temperature Uncertainty, uT

Source	Type	Uncertainty [°C]
$uT_{s[meas]}$	A	0.023
$uT_{s[res]}$	B	0.003
$uT_{s[std]}$	B	0.006
$uT_{s[self\ heating]}$	B	0.005
$uT_{s[thermal\ lag]}$	B	0.010
$uT_{s[gradients]}$	B	0.005
$uT_{s[control\ stability]}$	B	0.020
uT_s		0.033

3.4 Saturation Efficiency,

All two-pressure two-temperature humidity generators of single pass design rely on the ability of the saturator to fully saturate the gas with water vapor as it passes from inlet to outlet. Based on the counter flow design of the saturator (fully discussed in *Thermal Gradients*), it is assumed for all practical purposes that the saturator efficiency is 100%. Even given that assumption, small differences in saturation within the system can lead to uncertainty in the generated dew and frost point temperatures. This uncertainty is very small and very difficult to isolate or even measure outright, making it difficult to accurately quantify its value. Because of this, an instrument comparison was used to help identify this uncertainty. The premise of the instrument comparison was an RH Systems 373 chilled mirror. Data gathered during instrument comparison tests from several years of calibration history resulted in an estimated 99.95% efficiency. Table 6 shows this estimated efficiency in terms of dew and frost point temperature uncertainty.

Table 6. Saturation Efficiency Uncertainty, uSE

Nominal FP [°C]	uSE [°C]
-95	0.003
-90	0.003
-80	0.003
-70	0.003
-60	0.004
-50	0.004
-40	0.004
-30	0.005
-20	0.005
-10	0.006
0	0.006

3.5 Vapor Pressure

Vapor pressure equations are given in equations 1 and 2 for water and ice respectively. There are uncertainties assigned to the experimental data used as a basis for those equations. When calculating dew or frost point temperature from the humidity generator, the vapor pressure equations are only used as a means to transform data from saturation temperature to vapor pressure, then from vapor pressure back to dew or frost point temperature. Since the vapor pressure equations are used simply as bi-directional transfer functions, and round-trip transformation is always made, uncertainty in the data used to create these functions is of little significance, especially when the saturation temperature and dew or frost temperature are nearly equal. Under that circumstance, the dew and frost point vapor pressure is nearly equal to the saturation vapor pressure. However, when the ratio of saturation temperature to dew or frost temperature is high, the dew and frost point vapor pressure is lower than the saturation vapor pressure (likewise, the dew or frost point temperature is lower than the saturation temperature). This results in the possibility that the transformation function in one direction (temperature to saturation vapor pressure) occurs on a different portion of the function's curve than the inverse transformation (vapor pressure to dew or frost point temperature). Due to slight variations of the original experimental data, the quality of fit of the function to the data, and other factors, some allowance must be made for uncertainty associated with use of the vapor pressure equations. Table 7 shows this estimated vapor pressure uncertainty in terms of dew and frost point temperature.

Table 7. Vapor Pressure Uncertainty, *uSE*

Nominal FP [°C]	Ts [°C]	Ps [kPa]	uVp [°C]
-95	-80	1669.85	0.028
	-79.05	2000.00	0.028
-90	-80	597.27	0.029
	-75	1414.74	0.027
	-73.04	2000.00	0.027
-80	-80	101.33	0.030
	-72.37	344.74	0.029
	-70	498.10	0.028
	-60.91	2000.00	0.026
-70	-70	101.33	0.029
	-61.54	344.74	0.027
	-60	427.27	0.026
	-50	1667.30	0.024
	-48.66	2000.00	0.024
-60	-60	101.33	0.027
	-50.66	344.74	0.025
	-50	374.63	0.024
	-40	1277.80	0.023
	-36.28	2000.00	0.022
-50	-50	101.33	0.025
	-40	334.27	0.022
	-39.73	344.74	0.022
	-30	1020.10	0.021
	-23.76	2000.00	0.019
-40	-40	101.33	0.022

	-30	302.60	0.020
	-28.76	344.74	0.020
	-20	839.95	0.018
	-11.1	2000.00	0.016
-30	-30	101.33	0.019
	-20	277.21	0.017
	-17.73	344.74	0.016
	-10	708.82	0.015
	0	1723.90	0.014
	1.96	2000.00	0.014
-20	-20	101.33	0.015
	-10	256.50	0.012
	-6.66	344.74	0.011
	0	610.31	0.011
	10	1249.00	0.011
	16.91	2000.00	0.011
-10	-10	101.33	0.009
	0	239.37	0.006
	5.08	344.74	0.006
	10	484.44	0.006
	17	770.59	0.006
0	0	101.33	0.001
	10	204.24	0.001
	17	323.42	0.001
10	10	101.33	0.001
	17	160.17	0.002

3.6 Enhancement Factor

Enhancement factors are slight correction factors used to account for the non-ideal behavior of water vapor when admixed with other gases. The enhancement factor is dependent on both temperature and pressure and is given in equation 4. In use, the enhancement factor is applied as a multiplier to the vapor pressure resulting in the ‘effective vapor pressure’. For analytical purposes the enhancement factor can be considered in a manner similar to the vapor pressure for determining the affect of uncertainty in the enhancement factor data and formula. In the case of enhancement factor the pressure differences between saturator and test pressure results in the possibility that the transformation function in one direction occurs on a different portion of the function’s curve than the inverse transformation. Table 8 shows this estimated enhancement factor uncertainty in terms of dew and frost point temperature.

Table 8. Enhancement Factor Uncertainty, uEF

Nominal FP [°C]	Ts [°C]	Ps [kPa]	uEF [°C]
-95	-80	1669.85	0.045
	-79.05	2000.00	0.054
-90	-80	597.27	0.016
	-75	1414.74	0.037
	-73.04	2000.00	0.052
-80	-80	101.33	0.004

	-72.37	344.74	0.010
	-70	498.10	0.013
	-60.91	2000.00	0.047
-70	-70	101.33	0.004
	-61.54	344.74	0.009
	-60	427.27	0.010
	-50	1667.30	0.036
	-48.66	2000	0.043
-60	-60	101.33	0.004
	-50.66	344.74	0.008
	-50	374.63	0.009
	-40	1277.80	0.026
	-36.28	2000.00	0.040
-50	-50	101.33	0.004
	-40	334.27	0.008
	-39.73	344.74	0.008
	-30	1020.10	0.020
	-23.76	2000.00	0.037
-40	-40	101.33	0.004
	-30	302.60	0.007
	-28.76	344.74	0.008
	-20	839.95	0.016
	-11.1	2000.00	0.034
-30	-30	101.33	0.003
	-20	277.21	0.006
	-17.73	344.74	0.007
	-10	708.82	0.013
	0	1723.90	0.016
	1.96	2000.00	0.019
-20	-20	101.33	0.003
	-10	256.50	0.005
	-6.66	344.74	0.006
	0	610.31	0.007
	10	1249.00	0.011
	16.91	2000.00	0.017
-10	-10	101.33	0.002
	0	239.37	0.003
	5.08	344.74	0.004
	10	484.44	0.005
	17	770.59	0.007
0	0	101.33	0.002
	10	204.24	0.003
	17	323.42	0.004
10	10	101.33	0.002
	17	160.17	0.003

3.7 Permeation, Adsorption and Desorption

Permeation, adsorption and desorption refers to a continuous influx from or outgas to the humidity of the surrounding environment (such as the air within the laboratory) through small leaks or semi-permeable surfaces through the walls, fittings, valves, and dead spaces within the system. Although somewhat difficult to accurately quantify, instrument comparison has shown that these permeation affects start to become noticeable at frost point temperatures below approximately $-60\text{ }^{\circ}\text{C}$ with the greatest negative impact at the lowest frost point values. In the case of this generator, permeation affects are due to permeation of water vapor from the high ambient room conditions, through the tubing, fittings, or valves, into the dry gas stream output of the generator. Permeation tends to increase the frost point temperature of the generated gas stream. It is estimated that permeation tends to increase the concentration of water vapor in the gas stream independent of saturation temperature, saturation pressure, and test pressure. However, the effect of this permeation is most noticeable when running at the lowest frost points (where the concentration of water vapor in the gas is also very low), and at low flow rates. By increasing the flow rate of the generated gas, while assuming a constant permeation rate, the affect of that permeation is minimized. Like wise, if the gas being generated is of higher concentration (i.e., warmer frost point), then the affect of added water vapor from permeation is reduced. While the nominal frost point value being generated may be very low, it is recommended that the highest flow rate possible be used for the low frost point conditions to minimize the affect of permeation. This analysis assumes that the generator is run at the highest possible flow rate. Even with the high flow rate, permeations impact the generation of low frost point values. This permeation uncertainty is again very small and very difficult to isolate and measure outright, making it difficult to accurately quantify. Because of this, the same instrument comparison described in section 3.4 was also used to identify the uncertainty caused by permeation. Permeation leads more to an uncorrected bias than to a random variation in output, resulting in an offset during the instrument comparison tests. This offset was estimated to be five parts per billion (ppb). Table 9 shows the estimated permeation uncertainty bias in terms of frost point temperature.

Table 9. Permeation, absorption and desorption uncertainty bias, u_{bias}

Nominal [$^{\circ}\text{C}$]	bias [$^{\circ}\text{C FP}$]
-95	0.736
-90	0.292
-80	0.056
-70	0.013
-60	0.003
-50	0.001
-40	0.000

Because permeation leads more to an uncorrected bias than to a random variation in output it will be treated slightly different in the analysis. Uncertainty associated with uncorrected bias will be algebraically added to the expanded uncertainty rather than statistically included in combined uncertainty.

4. Combined Standard Uncertainty

The combined standard uncertainty, u_c , is obtained by statistical combination of the individual uncertainty components of pressure, temperature, and others, each multiplied by their associated sensitivity coefficients. Statistical combination is performed in accordance with the following:

$$u_c^2 = u_{c1}^2 + u_{c2}^2 + \dots + u_{cn}^2$$

where $u_{c1}, u_{c2}, \dots, u_{cn}$ are individual components of uncertainty each multiplied by their respective sensitivity coefficients

The following tables reflect the standard uncertainty components and the combined standard uncertainty at various frost and dew point temperatures at a variety of saturation pressures and temperatures.

Table 10. Combined Uncertainty at 10°C Dew Point

Combined Uncertainty at 10°C Dew Point						
Source	Name	Assigned Uncertainty	$T_s = 10.00$		$T_s = 17.00$	
			$P_s = 101.33$		$P_s = 160.19$	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.147	0.011	0.147	0.011
Sat Pressure <345	uP_s	0.076	0.147	0.011	0.093	0.007
Sat Pressure >345		0.304				
Sat Temp	uT_s	0.033	1.000	0.033	0.946	0.032
Vapor Pressure	uVp			0.001		0.002
Enhancement Factor	uEF			0.002		0.003
Saturation Efficiency	uSE			0.008		0.008
combined	u_c			0.038		0.035

Table 11. Combined Uncertainty at 0°C Dew Point

Combined Uncertainty at 0°C Dew Point								
Source	Name	Assigned Uncertainty	$T_s = 0.00$		$T_s = 10.00$		$T_s = 17.00$	
			$P_s = 101.33$		$P_s = 204.24$		$P_s = 323.42$	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.135	0.010	0.135	0.010	0.135	0.010
Sat Pressure <345	uP_s	0.076	0.135	0.010	0.067	0.005	0.042	0.003
Sat Pressure >345		0.304						
Sat Temp	uT_s	0.033	1.000	0.033	0.922	0.031	0.872	0.029
Vapor Pressure	uVp			0.001		0.001		0.001
Enhancement Factor	uEF			0.002		0.003		0.004
Saturation Efficiency	uSE			0.006		0.006		0.006
combined	u_c			0.037		0.033		0.032

Table 12. Combined Uncertainty at -10°C Frost Point

Combined Uncertainty at -10°C Frost Point														
Source	Name	Assigned Uncertainty	Ts = -10.00		Ts = 0.00		Ts = 5.08		Ts = 5.08		Ts = 10.00		Ts = 17.00	
			Ps = 10133		Ps = 239.37		Ps = 344.74		Ps = 344.74		Ps = 484.44		Ps = 770.59	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.111	0.008	0.111	0.008	0.111	0.008	0.111	0.008	0.111	0.008	0.111	0.008
Sat Pressure <345	uP_s	0.076	0.111	0.008	0.047	0.004	0.032	0.002						
Sat Pressure >345		0.304							0.032	0.010	0.023	0.007	0.014	0.004
Sat Temp	uT_s	0.033	1.000	0.033	0.872	0.029	0.784	0.026	0.784	0.026	0.754	0.025	0.713	0.024
Vapor Pressure	uVp			0.009		0.006		0.006		0.006		0.006		0.006
Enhancement Factor	uEF			0.002		0.003		0.004		0.004		0.005		0.007
Saturation Efficiency	uSE			0.006		0.006		0.006		0.006		0.006		0.006
combined	u_c			0.037		0.032		0.029		0.031		0.029		0.028

Table 13. Combined Uncertainty at -20°C Frost Point

Combined Uncertainty at -20°C Frost Point																
Source	Name	Assigned Uncertainty	Ts = -20.00		Ts = -10.00		Ts = -6.66		Ts = -6.66		Ts = 0.00		Ts = 10.00		Ts = 16.91	
			Ps = 10133		Ps = 256.50		Ps = 344.74		Ps = 344.74		Ps = 610.31		Ps = 1249.0		Ps = 2000.0	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.102	0.008	0.102	0.008	0.102	0.008	0.102	0.008	0.102	0.008	0.102	0.008	0.102	0.008
Sat Pressure <345	uP_s	0.076	0.102	0.008	0.040	0.003	0.030	0.002								
Sat Pressure >345		0.304							0.030	0.009	0.017	0.005	0.008	0.002	0.005	0.001
Sat Temp	uT_s	0.033	1.000	0.033	0.925	0.031	0.901	0.030	0.901	0.030	0.805	0.027	0.695	0.023	0.657	0.022
Vapor Pressure	uVp			0.015		0.012		0.011		0.011		0.011		0.011		0.011
Enhancement Factor	uEF			0.003		0.005		0.006		0.006		0.007		0.011		0.017
Saturation Efficiency	uSE			0.005		0.005		0.005		0.005		0.005		0.005		0.005
combined	u_c			0.039		0.035		0.034		0.035		0.031		0.030		0.031

Table 14. Combined Uncertainty at -30°C Frost Point

Combined Uncertainty at -30°C Frost Point																
Source	Name	Assigned Uncertainty	Ts = -30.00		Ts = -20.00		Ts = -17.73		Ts = -17.73		Ts = -10.00		Ts = 0.00		Ts = 196	
			Ps = 10133		Ps = 277.21		Ps = 344.74		Ps = 344.74		Ps = 708.82		Ps = 1723.9		Ps = 2000.0	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.094	0.007	0.094	0.007	0.094	0.007	0.094	0.007	0.094	0.007	0.094	0.007	0.094	0.007
Sat Pressure <345	uP_s	0.076	0.094	0.007	0.034	0.003	0.027	0.002								
Sat Pressure >345		0.304							0.027	0.008	0.013	0.004	0.005	0.002	0.004	0.001
Sat Temp	uT_s	0.033	1.000	0.033	0.922	0.031	0.905	0.030	0.905	0.030	0.851	0.028	0.738	0.025	0.682	0.023
Vapor Pressure	uVp			0.019		0.017		0.016		0.016		0.015		0.014		0.014
Enhancement Factor	uEF			0.003		0.006		0.007		0.007		0.013		0.016		0.019
Saturation Efficiency	uSE			0.005		0.005		0.005		0.005		0.005		0.005		0.005
combined	u_c			0.040		0.037		0.036		0.037		0.036		0.034		0.033

Table 15. Combined Uncertainty at -40°C Frost Point

Combined Uncertainty at -40°C Frost Point																
Source	Name	Assigned Uncertainty	Ts = -40.00		Ts = -30.00		Ts = -28.76		Ts = -28.76		Ts = -20.00		Ts = -11.10			
			Ps = 10133		Ps = 302.60		Ps = 344.74		Ps = 344.74		Ps = 839.95		Ps = 2000.0			
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert		
Test Pressure	uP_c	0.076	0.087	0.007	0.087	0.007	0.087	0.007	0.087	0.007	0.087	0.007	0.087	0.007		
Sat Pressure <345	uP_s	0.076	0.087	0.007	0.029	0.002	0.025	0.002								
Sat Pressure >345		0.304							0.025	0.008	0.010	0.003	0.004	0.001		
Sat Temp	uT_s	0.033	1.000	0.033	0.919	0.031	0.909	0.030	0.909	0.030	0.846	0.028	0.785	0.026		
Vapor Pressure	uVp			0.022		0.020		0.020		0.020		0.018		0.016		
Enhancement Factor	uEF			0.004		0.007		0.008		0.008		0.016		0.034		
Saturation Efficiency	uSE			0.004		0.004		0.004		0.004		0.004		0.004		
combined	u_c			0.041		0.038		0.038		0.038		0.038		0.046		

Table 16. Combined Uncertainty at -50°C Frost Point

Combined Uncertainty at -50°C Frost Point														
Source	Name	Assigned Uncertainty	Ts = -50.00		Ts = -40.00		Ts = -39.73		Ts = -39.73		Ts = -30.00		Ts = -23.76	
			Ps = 10133		Ps = 334.27		Ps = 344.74		Ps = 344.74		Ps = 1020.1		Ps = 2000.0	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.079	0.006	0.079	0.006	0.079	0.006	0.079	0.006	0.079	0.006	0.079	0.006
Sat Pressure <345	uP_s	0.076	0.079	0.006	0.024	0.002	0.023	0.002						
Sat Pressure >345		0.304						0.023	0.007	0.008	0.002	0.004	0.001	
Sat Temp	uT_s	0.033	1.000	0.033	0.915	0.031	0.913	0.030	0.913	0.030	0.839	0.028	0.794	0.026
Vapor Pressure	uVp			0.025		0.022		0.022		0.022		0.021		0.019
Enhancement Factor	uEF			0.004		0.008		0.008		0.008		0.020		0.037
Saturation Efficiency	uSE			0.004		0.004		0.004		0.004		0.004		0.004
combined	u_c			0.043		0.039		0.039		0.040		0.041		0.050

Table 17. Combined Uncertainty at -60°C Frost Point

Combined Uncertainty at -60°C Frost Point														
Source	Name	Assigned Uncertainty	Ts = -60.00		Ts = -50.66		Ts = -50.66		Ts = -50.00		Ts = -40.00		Ts = -36.28	
			Ps = 10133		Ps = 344.74		Ps = 344.74		Ps = 374.63		Ps = 1277.8		Ps = 2000.0	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.072	0.005	0.072	0.005	0.072	0.005	0.072	0.005	0.072	0.005	0.072	0.005
Sat Pressure <345	uP_s	0.076	0.072	0.005	0.021	0.002								
Sat Pressure >345		0.304					0.021	0.006	0.019	0.006	0.005	0.002	0.003	0.001
Sat Temp	uT_s	0.033	1.000	0.033	0.917	0.031	0.917	0.031	0.912	0.030	0.832	0.028	0.803	0.027
Vapor Pressure	uVp			0.027		0.025		0.025		0.024		0.023		0.022
Enhancement Factor	uEF			0.004		0.008		0.008		0.009		0.026		0.040
Saturation Efficiency	uSE			0.004		0.004		0.004		0.004		0.004		0.004
combined	u_c			0.044		0.041		0.041		0.041		0.045		0.053

Table 18. Combined Uncertainty at -70°C Frost Point

Combined Uncertainty at -70°C Frost Point														
Source	Name	Assigned Uncertainty	Ts = -70.00		Ts = -61.54		Ts = -61.54		Ts = -60.00		Ts = -50.00		Ts = -48.66	
			Ps = 10133		Ps = 344.74		Ps = 344.74		Ps = 427.27		Ps = 1667.3		Ps = 2000.0	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.066	0.005	0.066	0.005	0.066	0.005	0.066	0.005	0.066	0.005	0.066	0.005
Sat Pressure <345	uP_s	0.076	0.066	0.005	0.019	0.001								
Sat Pressure >345		0.304					0.019	0.006	0.015	0.005	0.004	0.001	0.003	0.001
Sat Temp	uT_s	0.033	1.000	0.033	0.921	0.031	0.921	0.031	0.907	0.030	0.823	0.027	0.812	0.027
Vapor Pressure	uVp			0.029		0.027		0.027		0.026		0.024		0.024
Enhancement Factor	uEF			0.004		0.009		0.009		0.010		0.036		0.043
Saturation Efficiency	uSE			0.003		0.003		0.003		0.003		0.003		0.003
combined	u_c			0.045		0.042		0.042		0.042		0.052		0.056

Table 19. Combined Uncertainty at -80°C Frost Point

Combined Uncertainty at -80°C Frost Point														
Source	Name	Assigned Uncertainty	Ts = -80.00		Ts = -72.37		Ts = -72.37		Ts = -70.00		Ts = -60.91			
			Ps = 10133		Ps = 344.74		Ps = 344.74		Ps = 498.10		Ps = 2000.0			
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.060	0.005	0.060	0.005	0.060	0.005	0.060	0.005	0.060	0.005		
Sat Pressure <345	uP_s	0.076	0.060	0.005	0.017	0.001								
Sat Pressure >345		0.304					0.017	0.005	0.012	0.004	0.003	0.001		
Sat Temp	uT_s	0.033	1.000	0.033	0.925	0.031	0.925	0.031	0.903	0.030	0.821	0.027		
Vapor Pressure	uVp			0.030		0.029		0.029		0.028		0.026		
Enhancement Factor	uEF			0.004		0.010		0.010		0.013		0.047		
Saturation Efficiency	uSE			0.003		0.003		0.003		0.003		0.003		
combined	u_c			0.046		0.043		0.044		0.043		0.060		

Table 20. Combined Uncertainty at -90°C Frost Point

Combined Uncertainty at -90°C Frost Point								
Source	Name	Assigned Uncertainty	Ts = -80.00		Ts = -75.00		Ts = -73.04	
			Ps = 597.03		Ps = 1414.7		Ps = 2000.0	
			Sens Coef	Std Uncert	Sens Coef	Std Uncert	Sens Coef	Std Uncert
Test Pressure	uP_c	0.076	0.054	0.004	0.054	0.004	0.054	0.004
Sat Pressure <345	uP_s	0.076						
Sat Pressure >345		0.304	0.009	0.003	0.003	0.001	0.002	0.001
Sat Temp	uT_s	0.033	0.897	0.030	0.849	0.028	0.829	0.028
Vapor Pressure	uVp			0.029		0.027		0.027
Enhancement Factor	uEF			0.016		0.037		0.052
Saturation Efficiency	uSE			0.003		0.003		0.003
combined	u_c			0.045		0.054		0.064

Table 21. Combined Uncertainty at -95°C Frost Point

Combined Uncertainty at -95°C Frost Point								
Source	Name	Assigned Uncertainty	Ts = -80.00		Ts = -79.05			
			Ps = 1668.9		Ps = 2000.0			
			Sens Coef	Std Uncert	Sens Coef	Std Uncert		
Test Pressure	uP_c	0.076	0.051	0.004	0.051	0.004		
Sat Pressure <345	uP_s	0.076						
Sat Pressure >345		0.304	0.003	0.001	0.002	0.001		
Sat Temp	uT_s	0.033	0.843	0.028	0.834	0.028		
Vapor Pressure	uVp			0.028		0.028		
Enhancement Factor	uEF			0.045		0.054		
Saturation Efficiency	uSE			0.003		0.003		
combined	u_c			0.060		0.067		

5 Expanded Uncertainty

Utilizing a coverage factor $k=2$, the expanded uncertainty, U , is computed using the formula

$$U = (k * u_c) + \text{bias}$$

Where k = the coverage factor (2 for 95% confidence level)

u_c = uncertainties at specific saturation temperatures and saturation pressures. u_c is obtained from the tables listed in section 4.

bias = any uncorrected bias associated with the nominal frost point or at a specific saturation temperature and saturation pressure. Values of bias are obtained from the table in section 3.7

Like the combined uncertainties listed in the tables of section 4, there are two separate calculations of expanded uncertainty at the saturation pressure of 344.74 kPa for many of the nominal frost point values.

This saturation pressure is at the switch point between use of the low-pressure transducer and use of the high-pressure transducer. At this saturation pressure, either transducer may be in use. If the low range transducer is in use, the associated expanded uncertainty is lower. If the high range transducer has switched in, then the expanded uncertainty will be larger in value. There is a step change (or more precisely, a discontinuity) in the associated expanded uncertainty at this possible pressure switching point.

The expanded uncertainty values are listed in table 22. Expanded uncertainty values are shown in °C frost point (°C dew point for nominal generated values above 0 °C).

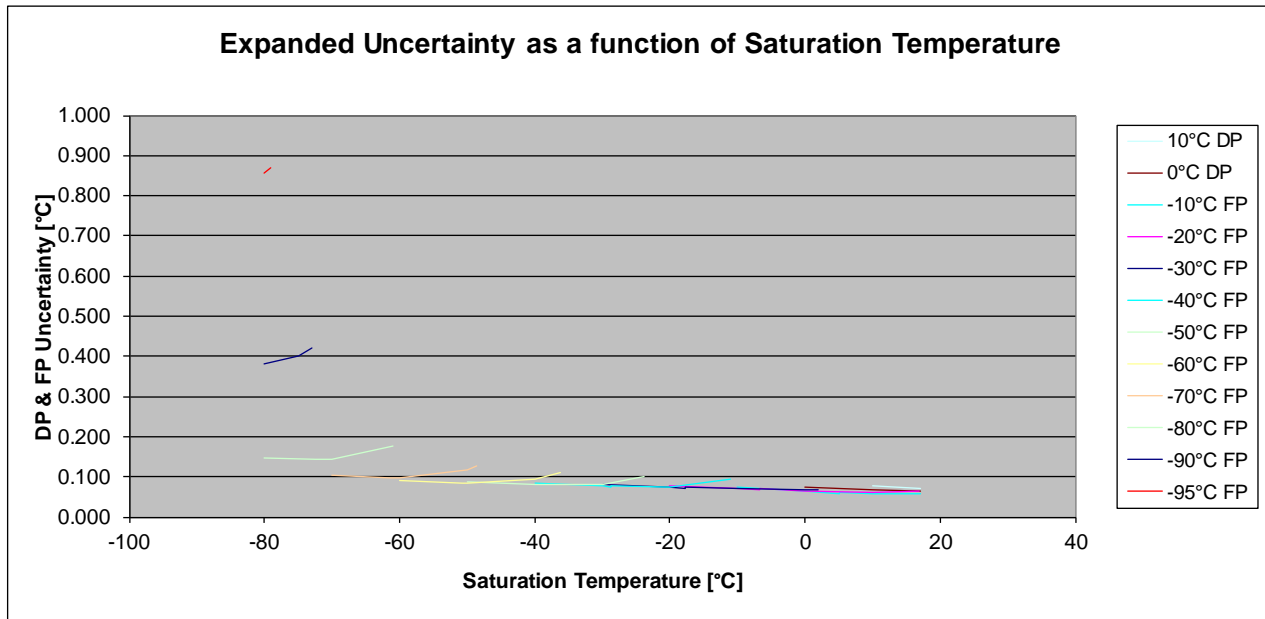
Table 22. Expanded Uncertainty (k=2)

Nominal FP [°C]	Ts [°C]	Ps [kPa]	U
-95	-80	1668.93	0.857
	-79.05	2000	0.870
-90	-80	597.029	0.382
	-75	1414.74	0.400
	-73.04	2000	0.421
-80	-80	101.325	0.148
	-72.37	344.74	0.143
	-72.37	344.74	0.144
	-70	498.099	0.143
	-60.91	2000	0.177
-70	-70	101.325	0.102
	-61.54	344.74	0.097
	-61.54	344.74	0.098
	-60	427.266	0.097
	-50	1667.27	0.117
	-48.66	2000	0.126
-60	-60	101.325	0.091
	-50.66	344.74	0.085
	-50.66	344.74	0.086
	-50	374.63	0.085
	-40	1277.76	0.093
-50	-36.28	2000	0.110
	-50	101.325	0.086
	-40	334.27	0.080
	-39.73	344.74	0.080
	-39.73	344.74	0.081
	-30	1020.07	0.082
-40	-23.76	2000	0.101
	-40	101.325	0.083
	-30	302.6	0.076
	-28.76	344.74	0.076
	-28.76	344.74	0.077
	-20	839.95	0.076
-30	-11.1	2000	0.093
	-30	101.325	0.080
	-20	277.21	0.073
	-17.73	344.74	0.072
	-17.73	344.74	0.074
	-10	708.82	0.071
	0	1723.92	0.067
-20	1.96	2000	0.067
	-20	101.325	0.078
	-10	256.5	0.071
	-6.66	344.74	0.069
	-6.66	344.74	0.071
	0	610.31	0.064
	10	1248.98	0.060

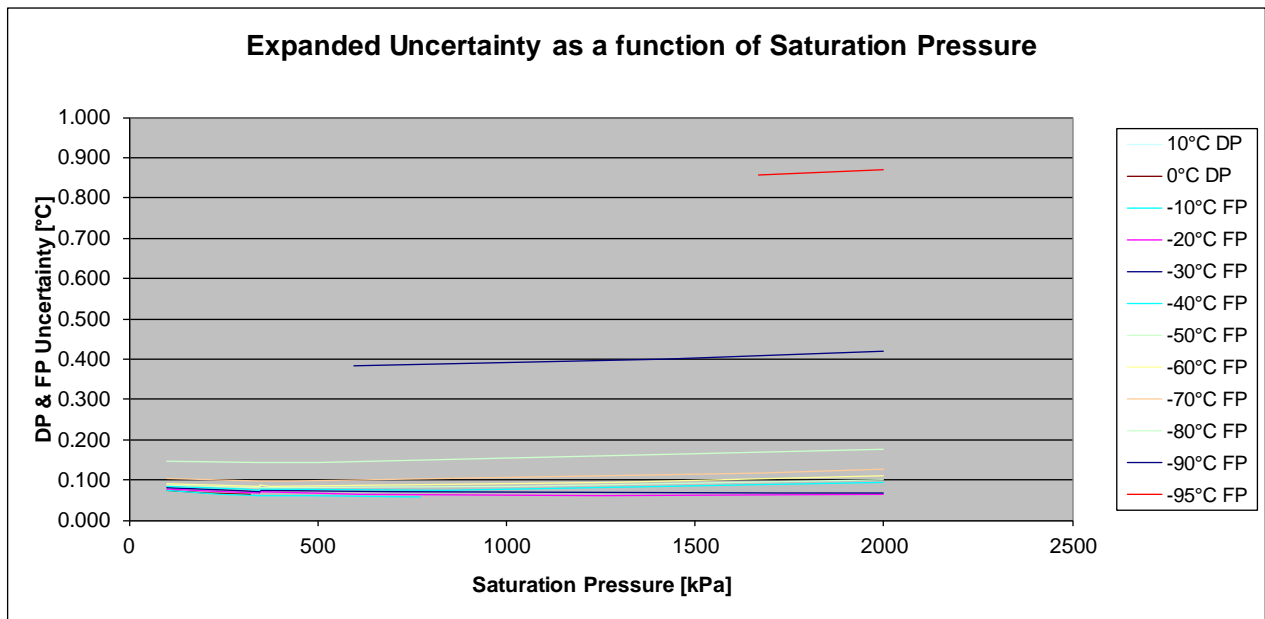
	16.91	2000	0.063
-10	-10	101.325	0.075
	0	239.37	0.065
	5.08	344.74	0.059
	5.08	344.74	0.062
	10	484.44	0.059
	17	770.59	0.057
0	0	101.325	0.075
	10	204.24	0.068
	17	323.42	0.065
10	10	101.325	0.076
	17	160.19	0.071

Note: 10 °C nominal frost point listed above is actually 10 °C dew point. Expanded uncertainties at this nominal dew point are also listed in °C dew point.

Graph 1: The following graph indicates the expanded uncertainties at various saturation pressures. Each nominal frost/dew point value is shown separately.



Graph 2: The following graph indicates the expanded uncertainties at various saturation temperatures.



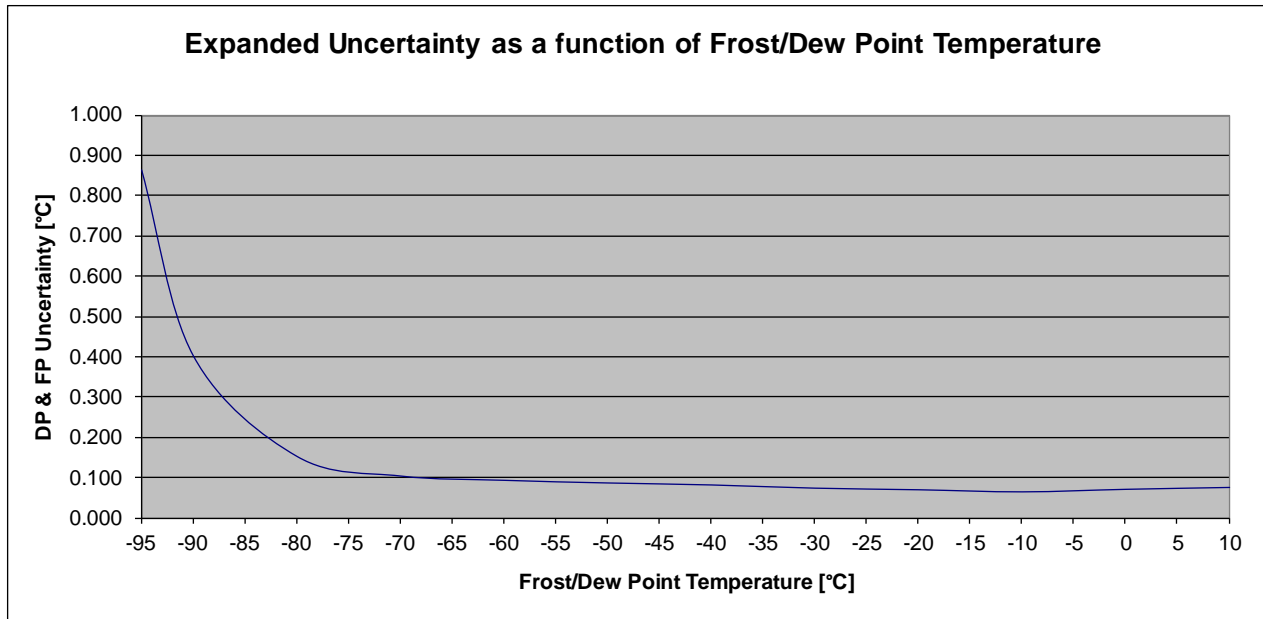
Each nominal frost/dew point value is shown separately.

Note that at each frost/dew point value, there is only small variation in the uncertainty at the various saturation temperature and saturation pressure combinations. In order to obtain uncertainty as a direct function of frost/dew point temperature, independent of the saturation temperature and pressure combinations, the average uncertainty value at each nominal frost/dew point value is chosen. The result of using the rounded average value of expanded uncertainty at each nominal frost/dew point is illustrated in the following table.

Table 23. Maximum Expanded Uncertainty (k=2)

Nominal FP [°C]	U
-95	0.9
-90	0.5
-80	0.2
-70	0.1
-60	0.1
-50	0.1
-40	0.1
-30	0.1
-20	0.1
-10	0.1
0	0.1
10	0.1

Graph 3: This graph, from the above table, depicts the maximum uncertainty at each of the nominal frost/dew point temperatures.



6 Summary

Expanded uncertainty of the low humidity generator in terms of dew point and frost point is relatively constant between +10 and about -75°C , varying only gradually over that entire range. For values below approximately -75°C , the affect of permeation tends to dominate the uncertainty.

It is also worth noting that the uncertainty at any given frost or dew point temperature is relatively constant regardless of the specific combinations of saturation temperature and saturation pressure. This allows the system to adequately generate frost or dew point temperatures without much regard for the specific saturation temperature-pressure combination chosen.

It is also noted that there was no assumed permeation, adsorption and desorption assigned to the RH Systems 373 chilled mirror comparison instrument. In reality the 373 has some quantifiable bias of permeation, adsorption and desorption for the very same reasons the 3900 does.

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9. A. Wexler and R.W. Hyland, “In Thermodynamic properties of dry air, moist air and water and SI psychrometric charts”, 1983 ASHRAE (Project 216–RP), Table 20.